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The *Geophysical Journal* is a journal for the publication of research in geophysics and related subjects. In addition to original papers, short notes and letters to the Editors are accepted for publication.

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Book reviews and reports of geophysical meetings are also published and the Editors are always glad to hear of such meetings and to receive short reports of them.

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Relative to the adopted value of 980·62200 cm/s^2 for the National Reference Station, the following gravity values have been obtained:

Winnipeg	980·9963
La Ronge	981·3949
Saskatoon	981·1356
Estevan	980·8598
Regina	980·9588

1. Introduction

Extensive gravity meter measurements were recently undertaken in Western Canada as part of a programme of co-operation with the oil industry for the revision of the regional gravity maps for the Prairie Provinces. The pendulum measurements described in this report provided a means of controlling the gravity meter surveys. The sites for the 1958 pendulum survey were as follows: Winnipeg, Man.; La Ronge, Sask.; Saskatoon, Sask.; Estevan, Sask.; Regina, Sask. These stations cover 535 mgal, virtually the entire range of the gravity meter observations.

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The final gravity values were obtained in two stages: (1) by the determination and adoption of a value for Winnipeg; (2) by the determination of values for the other stations on the basis of the value adopted for Winnipeg.

2. Apparatus and sites

Thompson (1959) has defined the terms used in reference to the apparatus; and has fully described the apparatus and allied equipment as well as the procedures used in operating and recording the observations.

Standardization was commenced in July 1958 at the Geophysical Laboratory of the Dominion Observatory in Ottawa preparatory to the transfer of operations to Winnipeg. From Winnipeg, observations were to be conducted over two loops: one running in a northerly direction to La Ronge and Saskatoon, the other westerly to Estevan and Regina. However, as is explained later, the return to Winnipeg from Regina had to be omitted. For all sites both groups of pendulums were used, three swings being completed for each pair of pendulums swung together, as illustrated in Table 2. The photographic records were scaled and the periods computed immediately after the completion of swings, so that any irregularities could be detected and remedied before leaving the site.

A new system of numbering the stations established with the new pendulum apparatus has been adopted, as follows:

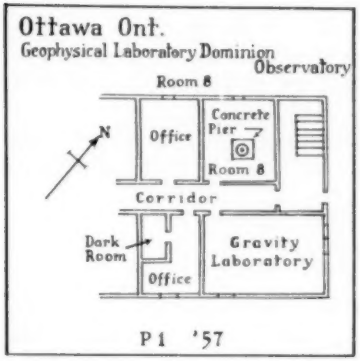
Ottawa	P_1–57	(Geophysical Laboratory)
Prescott	P_2–57	(R.C.A. Victor Plant)
Ithaca	P_3–57	(Cornell University)
Washington	P_4–57	(Commerce Building)
Vancouver	P_5–57	(University of British Columbia)
Winnipeg	P_6–57	(Dominion Building)
Kemptville	P_7–57	(Agricultural School)
Winnipeg	P_8–58	(University of Manitoba)
La Ronge	P_9–58	(School)
Saskatoon	P_{10}–58	(University of Saskatchewan)
Estevan	P_{11}–58	(Collegiate Institute)
Regina	P_{12}–58	(Museum of Natural History)

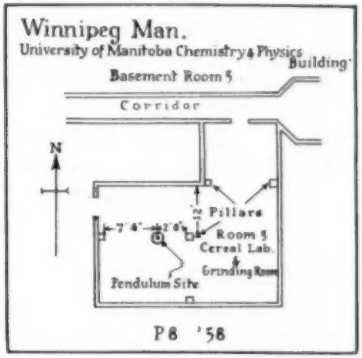
Stations P_1 to P_7–57 were observed in 1957 and have been described by Thompson (1959).

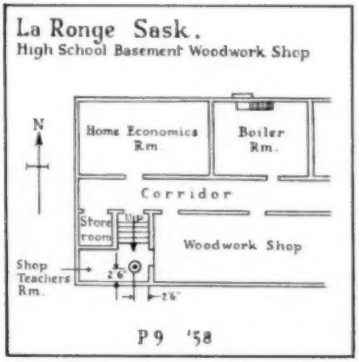
Descriptions of Sites of Pendulum Observations

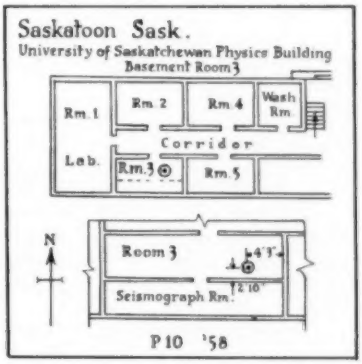
Winnipeg: P_8–58 University of Manitoba, Chemistry and Physics Building, Room No. 5, in basement. Site is on concrete floor, 7 ft 4 in. from west wall, 12 ft from north wall and 2 ft 8 in. from concrete pillar in centre of room.
Lat. 49° 49′ N Long 97° 08′ W (Elev. 756 ft).

La Ronge: P_9–58 La Ronge High School, woodwork shop in basement. Site is on concrete floor in storage room at west end of woodwork shop, 2 ft 6 in. from both the south wall and partition between storage room and workshop.
Lat. 55° 06′ N Long. 105° 17′ W (Elev. 1 228 ft).









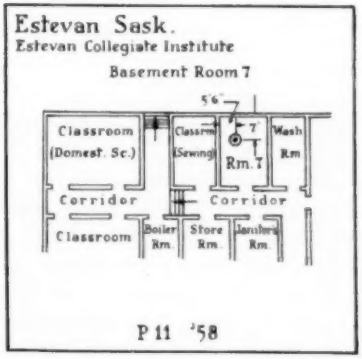


FIG. 1.—Plans of pendulum sites.
⊙: Position of apparatus.

Saskatoon: P_{10}–58 University of Saskatchewan, Physics Building, Room No. 3, in basement. Site is on concrete floor, 2 ft 10 in. from partition between Room No. 3 and seismograph room and 4 ft 3 in. from east wall. Lat. 52° 08′ N Long. 106° 38′ W (Elev. 1 631 ft).

Estevan: P_{11}–58 Estevan Collegiate Institute, Room No. 7, in basement. Site is on concrete floor, 5 ft 6 in. from west wall and 7 ft 0 in. from north wall. Lat. 49° 08′ N Long. 103° 00′ W (Elev. 1 868 ft).

Regina: P_{12}–58 Museum of Natural History, office at south end of corridor, main floor. Site is on concrete floor 4 ft 0 in. from west wall and 5 ft 5 in. from north wall. Lat. 50° 26′ N Long. 104° 37′ W (Elev. 1 877 ft).

3. The observations

The sequence of observations is given in Table 1. Throughout the sequence, a photographic record was made of the starting arcs of the pendulums. The resulting corrections to the pendulum periods were applied immediately after the swings to reduce them to the value for a 30 mm arc. The overall performance of the apparatus during this survey was consistent with its previous operation.

Table 1

Sequence of observations, 1958

Date	Station	Remarks
1958		
July 2–8	Ottawa (A_1)	
July 9–16	Ottawa (A_2)	
July 23–28	Winnipeg (A)	
Aug. 1–6	La Ronge	
Aug. 9–14	Saskatoon	
Aug. 18–23	Winnipeg (B)	
Aug. 26–31	Estevan	
Sept. 3–8	Regina	Pendulum case connected to 110 V line inadvertently.
Sept. 18–25	Ottawa (B)	Repairs completed. No change in operating point apparent.
Sept. 30–Oct. 5	Winnipeg (C)	
Oct. 27–Nov. 7	Ottawa (C)	Difficulties with inadequate line voltage.
Nov. 12–24	Ottawa (D)	Difficulties with inadequate line voltage.

There was no serious delay until Regina was reached. There while dismantling the apparatus, the operator inadvertently connected the 12 V line of the pendulum-case heaters to the 110 V power supply, thereby causing the heaters to become open-circuited. Fortunately the wire between the connectors of the heater windings was of low current rating and acted as a fuse, thus saving the heater windings from damage. It was felt, however, that for the necessary repairs it would be best to return to Ottawa where the equipment could be restandardized.

4. Computation of gravity values

Table 2 gives the periods, differences, means of pairs, variances, degrees of freedom and standard deviations for one set of swings known as Ottawa B, tabulated in the conventional manner.

Table 2

Sample of pendulum half-periods, 1958 (Ottawa B)

Set 2

Unit: 10^{-7} s

Date	S (Period)				ΔS (Period difference)			$\bar{S}$ (Mean period)		
	Pend. 4 K.E.1	Pend. 5 K.E.2	Pend. 5 K.E.1	Pend. 6 K.E.2	4–5	4–6	5–6	½(4+5)	½(4+6)	½(5+6)
1958 Sept. 18	4994482·9	4994443·8			39·1			4994463·4		
	4482·6	4441·6			41·0			4462·1		
	4485·1	4444·5			40·6			4464·8		
1958 Sept. 19	4481·1			4994438·7		42·4			4994459·9	
	4481·9			4438·8		43·1			4460·4	
	4480·3			4437·2		43·1			4458·8	
1958 Sept 20			4994494·9	4443·9			51·0			4994469·4
			4499·2	4441·1			58·1			4470·2
			4497·5	4439·6			57·9			4468·6
Mean	4994482·3	4994443·3	4994497·2	4994439·9	40·2	42·9	55·7	4994463·4	4994459·7	4994469·4
$\Sigma\rho^2$	13·89	4·58	9·38	27·47	2·01	0·33	32·69	3·65	1·34	1·28
Degrees of Freedom		14				6			6	
Variance		3·95				5·84			1·05	
Standard Deviation		1·99				2·42			1·02	

Note.—With one exception—Ottawa D, set 1—the same number of observations were conducted at each station, the degrees of freedom shown above apply. Complete tabulation of Observed Pendulum Periods may be obtained upon application to the Dominion Observatory, Ottawa, Canada.

$\Sigma\rho^2$—sum of squares of residuals.

The results of all the observations are summarized in Table 3. Examination of the mean periods of the observations at the Ottawa base indicates a considerable change between repeat observations. These have been plotted in Figure 2 and appear to be systematic rather than random. It may be noticed that there is a gradual decrease in period throughout the greater part of the observations. In view of this decrease it seemed preferable to abandon the customary method of determining differences in periods between stations and to assume that the periods change with time. The differences in half-period (ΔS) have therefore been taken directly from the apparent drift curves. It should be noted that the abrupt increase at Ottawa D, which is discussed later, does not affect the determinations of periods.

Differences of gravity (Table 4) have been computed, using the approximate relation:

$$\Delta g_1 = \frac{-2g_b}{S_b}\Delta S \qquad (1)$$

where g_b and S_b are the values of gravity and the pendulum half-period at the reference station and ΔS the difference in half-period between field station and reference station.

A second-order correction

$$\Delta g_2 = \frac{+3g_b}{S_b{}^2}(\Delta S)^2 \qquad (2)$$

has been applied to the mean values of the preliminary gravity differences. Final values of gravity are collected in Table 5. The value for Winnipeg, P_8–58, has been derived from the mean of the four ties obtained from the drift curves (Figure 2), and is based upon a value of 980·62295 cm/s^2 for the Ottawa pendulum station (Innes 1958). The values for La Ronge, Saskatoon, Estevan and Regina are relative to the Winnipeg value.

5. Accuracy of the measurements

The two main sources of errors in connection with the apparatus are drift in the frequency standard and in recording.

The method of employing the frequency standard is to beat and match the output from the standard with WWV's carrier, and experience has shown that the best beat frequency obtainable is 1 beat in 4 or 5 seconds. The beat frequency sometimes drifts erratically and can vary as much as 4 or 5 cycles in a half-hour period. When the 10 Mc/s carrier of WWV is used, this drift corresponds to a frequency shift of 5 parts in 10^7. Whilst the drift rate can be, and probably is, improved by fairly continuous matching with WWV, it is entirely dependent on the quality of reception obtainable during the observations. Some of the drift may be attributed to the effect of ionospheric disturbances; Garland & Cook (1955) have found that WWV can be received at Ottawa with an accuracy in the order of 0·4 parts in 10^7, whilst personnel at the Time Service Laboratory of the National Research Council claim that reception of WWV at Ottawa cannot be relied upon to better than 1 part in 10^7.

Recording errors may be placed in two groups: (1) errors introduced by recording the pendulum and timing dots on the photographic paper; (2) errors introduced by scaling the photographic records and converting the distances between dots into units of time.

Table 3

Summarized results of 1958 *pendulum periods: half-periods in* 10^{-7} s

Set 1

Date 1958	Station		Period (S) Pend. 1 K.E.1	Pend. 2 K.E.2	Pend. 2 K.E.1	Pend. 3 K.E.2	Differences (ΔS) 1–2	1–3	2–3	Mean Period ($\bar{S}$) ½(1+2)	½(1+3)	½(2+3)
5–8 July	Ottawa A_1	means	5004343·2	5004341·0	5004378·4	5004315·9	2·1	27·4	62·4	5004342·1	5004329·6	5004347·2
		s.d.		1·51				1·16			1·52	
14–16 July	Ottawa A_2	means	4343·5	4340·7	4380·3	4316·8	2·3	27·0	63·6	4341·9	4330·4	4348·5
		s.d.		1·26				1·64			1·00	
23–25 Sept.	Ottawa B	means	4339·6	4339·2	4376·8	4314·4	0·6	24·5	62·8	4339·5	4327·2	4345·4
		s.d.		1·25				1·73			0·97	
1–3 Nov.	Ottawa C	means	4337·9	4337·6	4381·7	4310·7	0·2	27·9	70·4	4337·7	4324·1	4346·6
		s.d.		1·63				2·46			1·20	
19–24 Nov.	Ottawa D	means	4342·4	4338·5	4383·1	4316·1	3·7	27·1	66·5	4340·4	4329·1	4349·9
		s.d.		1·83				2·92			1·22	
26–28 July	Winnipeg A	means	3391·4	3384·0	3427·3	3363·0	8·7	27·5	63·8	3388·4	3376·3	3395·4
		s.d.		3·66				7·30			1·20	
19–23 Aug.	Winnipeg B	means	3387·9	3386·6	3425·9	3363·0	0·2	26·9	62·0	3386·7	3375·6	3394·9
		s.d.		1·47				1·64			0·91	
3–5 Oct.	Winnipeg C	means	3385·7	3385·5	3424·2	3362·9	−0·3	22·0	62·6	3385·4	3375·1	3392·9
		s.d.		1·64				1·83			1·14	
4–6 Aug.	La Ronge	means	2371·8	2370·0	2410·4	2348·2	−0·1	25·3	62·2	2370·0	2360·9	2379·3
		s.d.		1·70				1·24			1·18	
12–14 Aug.	Saskatoon	means	3031·7	3031·9	3071·7	3007·7	−0·2	23·8	64·2	3031·8	3019·9	3039·6
		s.d.		1·61				3·08			0·79	
29–31 Aug.	Estevan	means	3736·6	3736·4	3771·9	3711·7	0·8	23·3	61·1	3736·8	3724·3	3741·3
		s.d.		1·28				1·52			0·82	
6–8 Sept.	Regina	means	3484·3	3481·3	3524·3	3458·6	1·2	27·0	66·1	3481·9	3472·6	3491·3
		s.d.		2·93				4·47			1·64	

Table 3

Summarized results of 1958 pendulum periods: half-periods in 10^{-7} s

Set 2

Date 1958	Station		Period (S) Pend. 4 K.E.1	Pend. 5 K.E.2	Pend. 5 K.E.1	Pend. 6 K.E.2	Difference (ΔS) 4–5	4–6	5–6	Mean period ($\bar{S}$) ½(4+5)	½(4+6)	½(5+6)
2–4 July	Ottawa A_1	means	4994487·4	4994445·6	4994501·3	4994443·5	41·0	45·0	57·6	4994466·0	4994465·8	4994472·5
		s.d.		1·61				1·43			1·46	
9–11 July	Ottawa A_2	means	4484·4	4448·1	4502·0	4445·1	35·0	41·1	56·6	4465·6	4465·2	4473·8
		s.d.		1·20				0·99			0·70	
18–20 Sept.	Ottawa B	means	4482·3	4443·3	4497·2	4439·9	40·2	42·9	55·7	4463·4	4459·7	4469·4
		s.d.		1·99				2·42			1·02	
27–31 Oct.	Ottawa C	means	4486·2	4438·9	4498·5	4438·4	47·6	43·1	64·5	4462·7	4464·3	4466·3
		s.d.		3·42				3·06			1·33	
12–18 Nov.	Ottawa D	means	4486·5	4446·9	4496·6	4442·6	39·3	45·0	53·2	4466·6	3364·2	4470·0
		s.d.		1·76				2·94			1·02	
23–25 July	Winnipeg A	means	3532·9	3493·9	3549·4	3490·9	38·1	44·2	57·3	3512·9	3511·8	3520·7
		s.d.		1·65				1·44			1·20	
20–22 Aug.	Winnipeg B	means	3532·4	3493·7	3549·5	3492·7	38·3	42·3	54·7	3512·8	3511·8	3522·1
		s.d.		2·38				3·16			1·34	
30–2 Oct.	Winnipeg C	means	3533·2	3492·3	3548·6	3491·4	39·6	42·0	58·3	3512·1	3513·5	3519·5
		s.d.		2·02				3·39			0·62	
1–3 Aug.	La Ronge	means	2519·8	2479·5	2534·5	2476·4	39·6	44·1	58·1	2499·4	2498·4	2505·4
		s.d.		2·82				5·06			1·62	
9–11 Aug.	Saskatoon	means	3178·2	3139·2	3198·1	3137·4	38·1	42·0	60·6	3158·2	3158·2	3167·8
		s.d.		1·32				1·33			1·06	
26–28 Aug.	Estevan	means	3878·9	3838·0	3899·2	3839·9	38·2	41·4	59·6	3857·2	3861·0	3869·4
		s.d.		2·04				1·05			1·58	
3–5 Sept.	Regina	means	3627·3	3587·2	3644·4	3587·2	40·3	40·5	56·6	3607·4	3606·9	3616·1
		s.d.		2·91				6·01			0·83	

Table 4

Mean half-periods and differences of gravity

Units: half-periods, 10^{-7}s, gravity, mgal

		Mean Period Set 1			Mean Period Set 2		
	Pendulums	1–2	1–3	2–3	4–5	4–6	5–6
First	Winnipeg	5003388·4	5003376·3	5003395·4	4993512·9	4993511·8	4993520·7
	Ottawa	5004341·5	5004329·8	5004347·9	4994465·2	4994464·0	4994472·9
	ΔS	−953·1	−953·5	−952·5	−952·3	−952·2	−952·2
	Δg	+373·53	+373·69	+373·29	+373·95	+373·91	+373·91
Second	Winnipeg	5003386·7	5003375·6	5003394·9	4993512·8	4993511·8	4993522·1
	Ottawa	5004340·7	5004328·8	5004346·8	4994464·3	4994461·8	4994471·2
	ΔS	−954·0	−953·2	−951·9	−951·5	−950·0	−949·1
	Δg	+373·88	+373·57	+373·06	+373·64	+373·05	+372·70
Third	Winnipeg	5003385·7	5003375·3	5003393·4	4993512·3	4993513·0	4993520·2
	Ottawa	5004339·5	5004327·2	5004345·4	4994463·4	4994459·7	4994469·4
	ΔS	−953·8	−951·9	−952·0	−951·1	−946·7	−949·2
	Δg	+373·80	+373·06	+373·10	+373·48	+371·75	+372·74
Fourth	Winnipeg	5003385·4	5003375·1	5003392·9	4993512·1	4993513·5	4993519·5
	Ottawa	5004339·0	5004326·4	5004345·7	4994463·2	4994460·8	4994468·4
	ΔS	−953·6	−951·3	−952·8	−951·1	−947·3	−948·9
	Δg	+373·72	+372·82	+373·41	+373·48	+371·99	+372·62

Mean Δg	+373·26 mgal
2nd-order correction	+ 0·11
Δg	+373·37
sd	0·59

Regina	5003481·9	5003472·6	5003491·3	4993607·4	4993606·9	4993616·1
Winnipeg	5003386·2	5003375·5	5003394·3	4993512·5	4993512·4	4993521·1
ΔS	+95·7	+97·1	+97·0	+94·9	+94·5	+95·0
Δg	−37·53	−38·08	−38·04	−37·29	−37·13	−37·33

Mean Δg	−37·57 mgal
2nd-order correction	0·00
Δg	−37·57
sd	0·40

Saskatoon	5003031·8	5003019·9	5003039·6	4993158·2	4993158·2	4993167·8
Winnipeg	5003387·2	5003375·8	5003395·1	4993512·8	4993511·8	4993521·5
ΔS	−355·4	−355·9	−355·5	354·6	−353·6	−353·7
Δg	+139·36	+139·56	+139·40	+139·33	+138·93	+138·97

Mean Δg	+139·26 mgal
2nd-order correction	+ 0·02
Δg	+139·28
sd	0·15

Table 4—*continued*

	Mean Period Set 1			Mean Period Set 2		
Pendulums	1–2	1–3	2–3	4–5	4–6	5–6
La Ronge	5002370·0	5002360·9	5002379·3	4992499·4	4992498·4	4992505·4
Winnipeg	5003387·8	5003376·0	5003395·3	4993512·8	4993511·8	4993521·1
ΔS	−1017·8	−1015·1	−1016·0	−1013·4	−1013·4	−1015·7
Δg	+399·11	+398·05	+398·41	+398·17	+398·17	+399·08

Mean Δg	+398·50 mgal
2nd-order correction	+ 0·12
Δg	+398·62
sd	0·50

Estevan	5003736·8	5003724·3	5003741·3	4993857·2	4993861·0	4993869·4
Winnipeg	5003386·4	5003375·5	5003394·6	4993512·6	4993512·1	4993521·5
ΔS	+350·4	+348·8	+346·7	+344·6	+348·9	+347·9
Δg	−137·40	−136·78	−135·95	−135·40	−137·09	−136·69

Mean Δg	−136·55 mgal
2nd-order correction	+ 0·01
Δg	−136·54
sd	0·74

Table 5

Values of gravity (cm/s^2)

Winnipeg P_8–58	980·99632 ± 0·00059 (Relative to Ottawa)
Regina P_{12}–58	980·95875 ± 0·00040 (Relative to Winnipeg, P_8–58)
Saskatoon P_{10}–58	981·13560 ± 0·00025 (Relative to Winnipeg, P_8–58)
La Ronge P_9–58	981·39494 ± 0·00050 (Relative to Winnipeg, P_8–58)
Estevan P_{11}–58	980·85978 ± 0·00074 (Relative to Winnipeg, P_8–58)

The errors in (1) are caused by inaccuracies in the rotation of the chronographic drum and shrinkage of the photographic paper during development. Thompson (1959) has shown that these errors are negligible. The errors in (2) are due primarily to the size of the dots on the records and the inability of an observer to judge their respective centres closer than 0·25 ms. Since two measurements, the start and the finish of a swing, are required to determine the period of a pendulum, the uncertainty in measuring the 3000 s interval is approximately 0·4 ms, or approximately 1 part in 10^7.

When pendulum 2 is swung on knife edge No. 1, its period is longer by $40{\cdot}3 \times 10^{-7}$ s; similarly, the period of pendulum 5 is longer by $56{\cdot}1 \times 10^{-7}$ s. This could be due either to differences in the knife edges themselves or to differences in the rigidity of the support for the knife edges. These variations appear to have a normal distribution about the values quoted, and there does not appear to be a systematic trend with time. This will be investigated further after uncertainties in the timing and recording equipment have been eliminated. Purchase of a more accurate frequency standard and development of an electronic timing system will make it possible to investigate these other sources of error.

The standard deviations for S, ΔS and $\bar{S}$ are listed in Table 3. With one exception—Ottawa D, set 1—all observations have the same number of degrees of freedom as the example shown in Table 2. The mean $\sigma(\Delta S)$ is 2·58 for set 1 and 2·69

for set 2; mean $\sigma(\bar{S})$ is 1·13 for set 1 and 1·15 for set 2. The ratio $\sigma(\Delta S)$ to $\sigma(\bar{S})$ is 2·3 to 1, somewhat larger than the value of 2 to 1 expected from error theory. In each set there are two unusually large standard deviations; rejecting these reduces the ratio to 1·9 to 1 indicating that there is no error correlation between the individual periods of the pendulum pairs. At Winnipeg A for set 1 $\sigma(\Delta S)$ is very large although $\sigma(\bar{S})$ is only slightly above average. This is clear evidence of ground movement. Similarly set 2 at La Ronge and both sets at Regina indicate ground movement.

6. Discussion of results

The most prominent feature of the drift curves of mean periods of pairs of pendulums swung together (Figure 2) is that all show what appears to be a systematic decrease in period with time. No suitable explanation for this can be offered. It might at first be supposed that the change is due to a gradual change in the properties of the thermistors which control the temperature of the case, but this is not borne out by the frequent relative calibrations of the individual thermistors, which are essentially the same as when first calibrated. It is extremely unlikely that the two types of thermistors employed age compatibly and thus give the same balance. Long-range observations of variation in periods may provide the answer.

Further examination of the drift curves at Ottawa reveals that the trend towards shorter periods was interrupted after the swings of pendulums 1 and 3 on 2 November (Ottawa C). Pendulums 2 and 3 swung on 3 November and pendulums 4 and 6 on 7 November, and all the combinations of pendulums in Ottawa D have periods longer by some 4 parts in 10^{-7} than would be expected if the downward trend had continued. This leads to the conclusion that some major event occurred on 2–3 November which greatly affected the apparatus. The records show that pendulum 4 received a bump on knife edge No. 2 on 28 October. Periods for pendulums 4 and 6 on 29 and 30 October were extremely erratic and were rejected. Periods for pendulums 5 and 6 on the 31st, for 1 and 2 on 1 November, and for 1 and 3 on 2 November followed the previous downward trend. The upward jump appears to have occurred prior to the swings of pendulums 2 and 3 that took place on 3 November. Since it seems highly improbable that the effect of the bump of 28 October could be obscured for three days and then show itself in the whole apparatus, we conclude that something else must have occurred. It was discovered at a later date that the output from the frequency standard was "motorboating" and the cause was found to be the failure of the screen-bypass capacitor in the final power amplifier that drives the synchronous motors of the chronograph. Since this output is not continuously monitored, it is possible that this condition was intermittent for some time, but whether it caused the observed changes in period is not known.

There is no evidence that one set of pendulums gives more accurate results than the other; therefore no weighting has been applied. In Table 6, comparison with work done previously at Winnipeg shows that agreement is good. Whilst the results obtained with the Cambridge apparatus came within 0·1 mgal, comparison with the determination given by the bronze-pendulum apparatus in 1957 shows a variation of 0·5 mgal. Possible reasons for this difference are that the site at P_6–57 is unstable and that the 1957 determination was based on 1 set of swings, whereas the 1958 determination is based on 6 sets. At the remaining sites, i.e. La

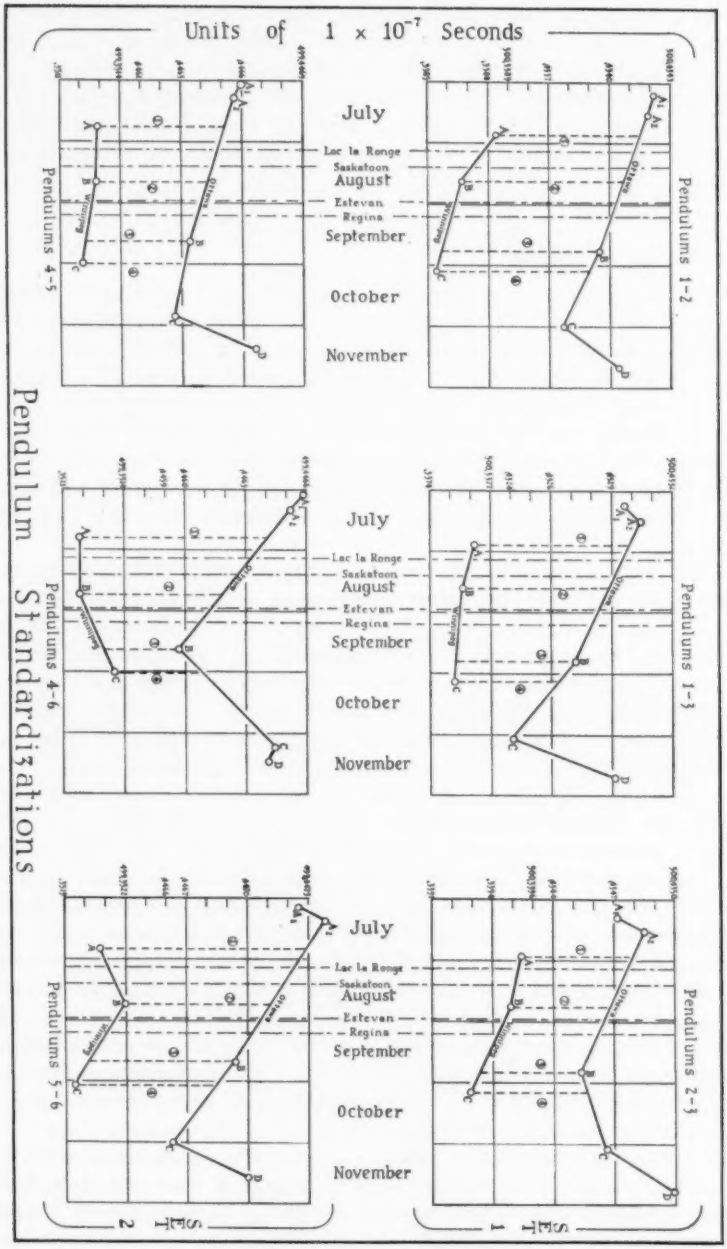


FIG. 2.—Drift curves of mean pendulum periods.

Ronge, Saskatoon, Estevan and Regina, previous pendulum observations were either not conducted or not connected.

Table 6

Comparison of values for Winnipeg

(Dominion Building P_6–57)

Apparatus	Observer	g (cm/s²)
Mendenhall apparatus	M. J. S. Innes 1946	980·994
Cambridge apparatus	G. D. Garland 1953	980·9952
Dominion Observatory Bronze	L. G. D. Thompson 1957	980·9948
Gravity meter	M. J. S. Innes L. G. D. Thompson 1952	980·9949
Dominion Observatory Bronze	P. J. Winter H. D. Valliant 1958	980·9953

Table 7 compares the gravity meter values (unpublished) at 4 pendulum sites on the basis of the values obtained at Winnipeg P_8–58. The agreement is good and, with the exception of the Saskatoon value, is within the expected deviation

Table 7

Comparison of results with gravity meter values

Pendulum Station	Values of gravity (cm/s²) pendulum	gravity meter	P−G mgal
Saskatoon	981·13560	981·13645	−0·85
Regina	980·95875	980·95885	−0·10
La Ronge	981·39494	981·39442	+0·52
Estevan	980·85978	980·85960	+0·18

(±0·4 mgal). The average difference is 0·4 mgal; the rms difference, however, is 0·5 mgal, which is much larger than that predicted by either method. The Winnipeg–Saskatoon difference must therefore be re-examined.

7. Acknowledgments

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Dominion Observatory,
Ottawa:
1959 *September.*

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On the Growth of Continents by Plastic Flow Under Gravity

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Summary

A coherent explanation of many major geological phenomena follows from the hypothesis that the strength of basement rocks at shallow levels is very small for long-continued stresses, and that gradual plastic flow occurs widely in a superficial layer of the crust. A likely mechanism of flow in a layer 2–3 km thick is that of solution and recrystallization.

By an adaptation of Nye's theory of flow in ice-sheets it is shown that the high-standing continental masses are approximately in equilibrium for a yield point of order 10^6 dyn/cm^2. Any increase in the average height of a continent by volcanic or orogenic activity causes the continents to grow in area and thus to encroach upon neighbouring oceans. The pattern of growth is in general asymmetrical and changes with time; recent rates of radial expansion are estimated to have ranged up to the order of 0·1 cm/yr. Such a process of growth by seaward flow of a superficial plastic layer is compatible with many actual features of continental margins; submarine canyons may have been formed by subaerial erosion and later carried across the coastline and submerged. Continental shields are regions that have remained unaffected by flow through several geological eras.

The configuration of the slip-line field in a plastic sheet suggests a mechanism of geological faulting by shear fracture induced by plastic flow. Rift valleys and tilted blocks are thus seen as essentially shallow phenomena. Thrust sheets and insular aprons are attributed to extrusion at the base of plastically subsiding mountains and sea-mounts. The fundamental postulate of stress tectonics, that the crust is acted upon by horizontal forces originating in the mantle, becomes largely redundant. Disturbances projected into the substratum as a result of superficial flow are linked with orogenic, volcanic and seismic activity in a complex of cause and effect. It is suggested that mid-oceanic elevations develop by deformation of the substratum, due to progressive loading at the adjacent margins as the continents expand.

1. Introduction

The origin and development of oceans and continents has been well called the central problem of geology. These most obvious features of the Earth's surface

have been the subject of a great variety of hypotheses but so far no generally acceptable theory has emerged; indeed, hardly any question of importance in this field can yet be regarded as beyond controversy. Was the crust at one time all oceanic—or all continental—or have there been oceans and continents since the Earth was in its infancy? Have continents become larger or smaller, or have they merely drifted about? It may be necessary to look into the mantle beneath for processes which could give rise to major disturbances of the surface; thus we have the hypotheses of thermal contraction and convection. But since it is not yet known whether the Earth is heating or cooling, this class of theory appears to lack a definite starting point.

Field studies of the crust by the methods of geology and geophysics are yielding an increasing number of important generalizations which can only lead to a narrowing of the present range of speculation. The distinction between continent and ocean is now clear; it appears that all the continents have an average crustal thickness close to 35 km, and the oceans 5–10 km (excluding the water). All the continents incorporate inert pre-Cambrian shields, and all the oceans have active median elevations. The well-known fact that the solid surface has two dominant levels—one centred at about 100 m above present sea-level and the other at about 4 700 m below it—is no longer the isolated piece of evidence that it was when Fisher first deduced from it the existence of two distinct types of crust.

The critical region for any theory of continents and oceans is the zone where these two types of crust meet, usually called the continental margin. The crustal section across this transition is complex and difficult to map, and this has been achieved only for limited portions of the east coast of North America and the west coast of Europe. The surface profile at the margin has almost everywhere, however, the same main features; the gently sloping continental shelf extends out to a depth of some 200 m, and here begins the relatively steep continental slope, by which the bottom level is carried down to that of the deep ocean floor. The shelf was formerly believed to be essentially an accumulation of erosional debris, but this view has had to be abandoned since dredging revealed outcrops of continental basement rock, including granite, at the outer edge of the shelf, and still more, since seismic sounding confirmed the continuity of the basement beneath the shelf sediments. A striking feature of the margin is the widespread occurrence of deep valleys, comparable in size with land canyons, extending oceanwards across shelf and slope.

Most of the Earth's zones of major crustal disturbance are situated at or near continental margins, though about half of the total length of margin appears to be stable. At the disturbed margins one finds a characteristic sequence of volcanic, orogenic and seismic activity, together with a tendency for ocean trenches to occur immediately offshore. The class of disturbance that occurs along the mid-ocean ridges is of a distinctly lesser magnitude, but nevertheless includes volcanoes, earthquakes and crustal rifts.

Many of these features have become fully established as of widespread occurrence only in the last few years, and much detail is still lacking. But it does appear that most of the salient characteristics of the outer surface and crust are now known, and in so far as the history of the oceans and continents has been involved with near-surface phenomena we are perhaps already in possession of the chief evidence upon which a comprehensive theory must be based.

Nevertheless, when we consider more closely the detailed topography of continent and ocean, we find that the sum of all processes known to geology leaves

many important features inadequately explained. Erosion and sedimentation, volcanic activity, and isostatic movement can all be directly observed, but beyond these one moves further into the realm of hypothesis. The great vertical irregularities—geosynclines and mountain ranges—are regarded as by-products of processes located in the interior; but it should be noted that the supposed forces which these internal processes exert on the crust—whether by contraction or convection—are primarily horizontal. In the study of tectonics it seems a matter of some hazard to define the stress system associated with any observed deformation, except where the basic force involved is gravity. It is remarkable that the rift valley, the normal block-fault and the ocean trench have each been ascribed now to compression, now to tension, and one may be led to ask whether the key to such geological phenomena as these might not lie in some different process altogether.

There is evidence of a possible further process of deformation, which may subject the upper layers of the crust to widespread horizontal movement, and with the help of isostasy may also cause localized vertical movement. This process is one of plastic flow actuated by gravity; it is similar to the process that governs the behaviour of ice-sheets, and we shall make some use of theory that has recently been developed in this connexion. Such movement may at first sight seem akin to gravitational gliding, but plastic flow under gravity requires neither an inclined base plane nor a lubricating layer. These are essential conditions for gravitational gliding, which de Sitter (1956) regards, moreover, as subsidiary to lateral compression, the glide-slope itself being a product of horizontal thrust. If these prerequisites could be set aside, many examples of large-scale folding, including nappe formation, might perhaps find a simpler explanation than hitherto. Indeed the process to be discussed here may add credence to the view of van Bemmelen (1950), that all horizontal deformations in structural geology are products of the force of gravity; though transcurrent faulting with large displacement could hardly be subsumed under this rule. Energy from non-gravitational sources is thus required primarily to achieve an upward transport of material, such as we witness in mountain-building and volcanic eruption.

2. Secular yielding of crustal rocks

The strength of crustal rocks is widely believed to be very great, and a careful examination of the grounds for this view must accompany any suggestion of widespread flow. The general argument has been summarized by Gutenberg (1941) as follows: "From the fact that mountains apparently have not changed their height appreciably in historic times, and also from laboratory experiments, it follows that in the uppermost layers (at least 30 km thick) the strength is of the order of 10^9 to 10^{10} dyn/cm^2." It will be shown that for continental basement rocks lying within a few kilometres of the surface the present hypothesis implies a yield point of very much smaller order of magnitude—about 10^6 dyn/cm^2. This huge discrepancy may simply be a measure of the difference of time scale between the phenomena involved in the usual discussions of strength, on the one hand, and the process of plastic flow on the other.

The existence of mountains and gravity anomalies shows only, as Jeffreys (1952, p. 200) has pointed out, that the crust can sustain large stress differences without immediate fracture. If gradual yielding does occur it must be at smaller stresses. Benioff & Gutenberg (1951), notwithstanding their belief in a generally strong crust, have pointed to observed folds in rocks as evidence that flow can

occur; they also emphasize that strength is a function of the time available, and even lend support to the view that "given enough time, no rock has any strength". A very different view is expressed by Daly (1951): "The persistence of the buckler [i.e. continent] relation proves that sialic rock has withstood shearing stress for 1 000 million years." Our present proposal lies somewhere between these two extremes: we shall show that the main features of the high-standing continental slabs may actually be ascribed to a yielding under shear stress; and although this yielding must have gone on more or less continually throughout the history of the continents, yet even on this time scale there is a definite value of shear stress below which appreciable yielding does not occur. It is a corollary of this hypothesis that mountains must slowly subside.

Can we expect to observe this subsidence of mountains? Perhaps the slowest persistent subsidence that has so far been recorded in geology, by whatever means, is that of the guyots and atolls, which appear to have sunk at an average rate of about 2 cm/1 000 yr (Kuenen 1954). This phenomenon cannot be attributed to a rising sea level (Hess 1955); and the movement is exceedingly slow compared with presumed isostatic effects—less than one-thousandth of the average rate of uplift in Fennoscandia since the ice began to melt. Woollard (1954) considers that oceanic volcanoes subside by elastic creep of the crust, but the uniformity of subsidence rate in the examples cited by Kuenen argues against this explanation. Kuenen himself, following Carsola & Dietz (1952) favours plastic yielding of the crust. One would expect, however, that the volcanic cone itself would yield more readily than the primaeval oceanic crust, if only because the lack of lateral constraint will locate the greatest stress difference in the lower portions of the volcano rather than in the crust. Beneath the deep ocean floor, moreover, conditions become inimical to plastic flow, as we shall see later. Finally, the gravity anomalies that have been found associated with sea-mounts suggest that such features are often isostatically uncompensated (Shurbet & Worzel 1955). Hence the subsidence of guyots and atolls may be attributed primarily to plastic spreading at the base, though other mechanisms may play some part.

The nature of the ocean floor in the neighbourhood of volcanic islands and guyots is strongly suggestive of the occurrence of plastic spreading. Often the steep sides of the central cone grade outwards into a smooth, gently sloping "apron" that may extend for many hundreds of kilometres (Menard 1956). Seismic refraction measurements in several of these regions, including the very extensive Bermuda Rise, have been interpreted as showing that the "apron" is formed of the same kind of volcanic rock as the islands themselves (Gaskell 1954). Molten lava from the central vent could hardly flow out to such great distances, however, and Menard (1956) has therefore postulated an additional source in the form of crustal fissures located around the base of the main volcano. In proposing secular plastic flow as an alternative explanation we look upon subsidence of the central cone and the development of an "apron" as two aspects of the same process of deformation. This process will continue long after the volcano has become extinct, until eventually the island is transformed into a thin sheet upon the floor of the ocean. By analogy with the spreading of continents, which will be discussed in detail below, we may estimate that a sheet with an equilibrium thickness of 1–1½ km at the centre will be several hundred kilometres in diameter. Pre-Cretaceous volcanoes will have largely reached this state by now, and since on present seismic evidence (Hill 1957) it seems possible that a thin layer of volcanic rock occurs rather widely in oceanic regions we may perhaps understand why

pre-Cretaceous sediments have so far eluded discovery in the Pacific Basin (Revelle, Bramlette, Arrhenius & Goldberg 1955).

Mountains on land are presumably subject to the same kind of yielding as mountains rising from the ocean floor, but for many reasons the movement will be harder to recognize. If we may assume about the same rate of movement we find that the total plastic subsidence of a mountain since the climax of the Alpine orogeny amounts to about 400 m. Such a diminution of height will scarcely be identifiable when obscured by the effects of erosion, and partially offset by isostasy. The more ancient mountain ranges have in fact been much reduced in height, and as Jeffreys (1952, pp. 318–320) has pointed out, this cannot be accounted for by denudation alone. Allowing for isostatic adjustment, a lowering by 3 km would require the removal of some 14 km from the surface; yet old sedimentary beds have survived the lowering process in ranges such as the Appalachians and the mountains of Wales and Scotland. Jeffreys shows that the facts could be explained by an outflow of some 20 km of deep crustal material, which could conceivably be in a condition close to melting. Our analogy with the guyots offers an alternative explanation: that the base of the mountains has flowed out over the surface of the adjacent lower-lying country. When the height of a plastic body is much greater than the equilibrium value, flow occurs chiefly at the base, whilst the upper portion merely subsides (Orowan 1949). Thus a mountain range is lowered by a process of extrusion which leaves the peaks and remoter valleys more or less intact; hence with erosion relegated to a minor role it is to be expected that some old sedimentary beds will be preserved. The isostatic adjustment called for in this case might be relatively small, depending upon the degree of regionality, since much of the extruded material would remain in the general neighbourhood. On our previous assumptions the gross amount of lowering by extrusion since the Caledonian orogeny could total about 6 km, which seems to be of the right order. The extruded material may well have formed the so-called "thrust sheets" that so often extend from the foot of a mountain range far into the adjacent lowland, and which we may regard as analogous to the layers of volcanic rock surrounding guyots and atolls in the ocean.

Now let us turn to laboratory experiments, the other line of evidence commonly adduced in favour of great crustal strength. The literature abounds with reports of deformation tests on specimens of rock. These tests fall into two main categories neither of which, curiously enough, has any great degree of relevance to our present problem. On one hand there is the class of experiment in which a dry uncovered specimen is subjected to simple compression, tension, shear or bending. On the other hand we find the high pressure techniques of P. W. Bridgman applied to rocks, especially in the U.S.A.; these experiments commonly involve confining pressures up to 10000 atm (corresponding to a depth of about 35 km), stress differences up to several times 10^9 dyn/cm^2 and temperatures up to several hundred degrees C. Clearly, in neither category do the experimental conditions simulate the environment of rocks in place near the surface of the crust.

The presumption of great strength has been based chiefly on results of the former type of experiment. A test that might appear rather convincing in this regard was carried on over a long period by D. Griggs, who reported on the progress of deformation from time to time. An unconfined cylinder of Solenhofen limestone, which has a crushing strength of $2{\cdot}5 \times 10^9$ dyn/cm^2, was subjected to a compressive stress of $1{\cdot}4 \times 10^9$ dyn/cm^2 (Griggs 1939). Deformation proceeded at a steadily diminishing rate, after the manner of elastic (i.e. recoverable) creep;

occur; they also emphasize that strength is a function of the time available, and even lend support to the view that "given enough time, no rock has any strength". A very different view is expressed by Daly (1951): "The persistence of the buckler [i.e. continent] relation proves that sialic rock has withstood shearing stress for 1 000 million years." Our present proposal lies somewhere between these two extremes: we shall show that the main features of the high-standing continental slabs may actually be ascribed to a yielding under shear stress; and although this yielding must have gone on more or less continually throughout the history of the continents, yet even on this time scale there is a definite value of shear stress below which appreciable yielding does not occur. It is a corollary of this hypothesis that mountains must slowly subside.

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The nature of the ocean floor in the neighbourhood of volcanic islands and guyots is strongly suggestive of the occurrence of plastic spreading. Often the steep sides of the central cone grade outwards into a smooth, gently sloping "apron" that may extend for many hundreds of kilometres (Menard 1956). Seismic refraction measurements in several of these regions, including the very extensive Bermuda Rise, have been interpreted as showing that the "apron" is formed of the same kind of volcanic rock as the islands themselves (Gaskell 1954). Molten lava from the central vent could hardly flow out to such great distances, however, and Menard (1956) has therefore postulated an additional source in the form of crustal fissures located around the base of the main volcano. In proposing secular plastic flow as an alternative explanation we look upon subsidence of the central cone and the development of an "apron" as two aspects of the same process of deformation. This process will continue long after the volcano has become extinct, until eventually the island is transformed into a thin sheet upon the floor of the ocean. By analogy with the spreading of continents, which will be discussed in detail below, we may estimate that a sheet with an equilibrium thickness of 1–1½ km at the centre will be several hundred kilometres in diameter. Pre-Cretaceous volcanoes will have largely reached this state by now, and since on present seismic evidence (Hill 1957) it seems possible that a thin layer of volcanic rock occurs rather widely in oceanic regions we may perhaps understand why

pre-Cretaceous sediments have so far eluded discovery in the Pacific Basin (Revelle, Bramlette, Arrhenius & Goldberg 1955).

Mountains on land are presumably subject to the same kind of yielding as mountains rising from the ocean floor, but for many reasons the movement will be harder to recognize. If we may assume about the same rate of movement we find that the total plastic subsidence of a mountain since the climax of the Alpine orogeny amounts to about 400 m. Such a diminution of height will scarcely be identifiable when obscured by the effects of erosion, and partially offset by isostasy. The more ancient mountain ranges have in fact been much reduced in height, and as Jeffreys (1952, pp. 318–320) has pointed out, this cannot be accounted for by denudation alone. Allowing for isostatic adjustment, a lowering by 3 km would require the removal of some 14 km from the surface; yet old sedimentary beds have survived the lowering process in ranges such as the Appalachians and the mountains of Wales and Scotland. Jeffreys shows that the facts could be explained by an outflow of some 20 km of deep crustal material, which could conceivably be in a condition close to melting. Our analogy with the guyots offers an alternative explanation: that the base of the mountains has flowed out over the surface of the adjacent lower-lying country. When the height of a plastic body is much greater than the equilibrium value, flow occurs chiefly at the base, whilst the upper portion merely subsides (Orowan 1949). Thus a mountain range is lowered by a process of extrusion which leaves the peaks and remoter valleys more or less intact; hence with erosion relegated to a minor role it is to be expected that some old sedimentary beds will be preserved. The isostatic adjustment called for in this case might be relatively small, depending upon the degree of regionality, since much of the extruded material would remain in the general neighbourhood. On our previous assumptions the gross amount of lowering by extrusion since the Caledonian orogeny could total about 6 km, which seems to be of the right order. The extruded material may well have formed the so-called "thrust sheets" that so often extend from the foot of a mountain range far into the adjacent lowland, and which we may regard as analogous to the layers of volcanic rock surrounding guyots and atolls in the ocean.

Now let us turn to laboratory experiments, the other line of evidence commonly adduced in favour of great crustal strength. The literature abounds with reports of deformation tests on specimens of rock. These tests fall into two main categories neither of which, curiously enough, has any great degree of relevance to our present problem. On one hand there is the class of experiment in which a dry uncovered specimen is subjected to simple compression, tension, shear or bending. On the other hand we find the high pressure techniques of P. W. Bridgman applied to rocks, especially in the U.S.A.; these experiments commonly involve confining pressures up to 10 000 atm (corresponding to a depth of about 35 km), stress differences up to several times 10^9 dyn/cm^2 and temperatures up to several hundred degrees C. Clearly, in neither category do the experimental conditions simulate the environment of rocks in place near the surface of the crust.

The presumption of great strength has been based chiefly on results of the former type of experiment. A test that might appear rather convincing in this regard was carried on over a long period by D. Griggs, who reported on the progress of deformation from time to time. An unconfined cylinder of Solenhofen limestone, which has a crushing strength of $2{\cdot}5 \times 10^9$ dyn/cm^2, was subjected to a compressive stress of $1{\cdot}4 \times 10^9$ dyn/cm^2 (Griggs 1939). Deformation proceeded at a steadily diminishing rate, after the manner of elastic (i.e. recoverable) creep;

eleven years later the total strain was 0·02 per cent (Griggs 1951). If the creep were to continue in the same manner for 50 million years the strain would amount to little more than 0·05 per cent (as compared with an observed 20 per cent for guyots). There is, of course, no actual warrant for any such extrapolation of laboratory results in time; indeed Birch (1955) has remarked that the difficulties in the way of relating experimental data to geological periods seem insuperable. Perhaps some degree of optimism may be entertained, however, in cases where the magnitude of stress, strain, and rate of strain are sufficiently small. Certainly the kind of experiment described above offers no encouragement for attributing large-scale geologic deformations to elastic creep; but in the light of other experimental results one may doubt whether the unconfined compression test has any real significance to geology.

The strength of a rock may be much reduced, as it happens, by providing an environment more nearly comparable to the natural one. Although few experimentalists have aimed at reproducing the natural conditions of rocks at shallow levels, enough has been done to indicate that one factor occurring in nature is of fundamental importance, i.e. the presence of water. Dry crystalline quartz exhibits no tendency to flow even when subjected to enormous stress differences—up to 40×10^9 dyn/cm^2 (Bridgman 1952)—but with water present quartz becomes mobile and quartz aggregates are readily deformable (Griggs 1940, 1951). The effect of water on a soluble solid under stress has been described from the standpoint of thermodynamics by Goranson (1940). Solubility is essentially anisotropic under directed stress, and there ensues an overall migration of solid away from the planes of maximum compressive stress. The magnitude of stress and the rate of strain are theoretically related in a manner identical to that found empirically by Griggs from experiments on gypsum.

The process of flow by solution and recrystallization may be fundamental to an explanation of plastic movement at shallow levels in the crust. In theory this type of deformation may occur at any stress, however small (Goranson 1940), and in this sense there is truth in the dictum of Benioff & Gutenberg (1951) quoted previously. But the rate of strain decreases very rapidly as the stress is reduced, so that in any given practical context a value of stress will be found below which flow is imperceptible. For his detailed study of plastic flow in the presence of a solvent, Griggs (1940) selected gypsum as being a common mineral which promised measurable recrystallization in the time available in the laboratory. He found that dry gypsum fractured, without having flowed appreciably, at a compressive stress of about 5×10^8 dyn/cm^2. When the specimen was in contact with its own aqueous solution, on the other hand, a steady deformation could be observed even at a stress of 1×10^8 dyn/cm^2. Thus, the strength was reduced by a factor of at least 5. Yet the lowest rate of strain that Griggs had the means of observing (about 7 millionths per day) was still nearly 1 million times greater than that deduced above for the case of guyots and atolls. And although Griggs considered, from certain details of his results, that the gypsum would entirely cease to flow if the stress were reduced a little further, it is more consistent with Goranson's theory to expect that comparatively tiny stresses would produce significant deformation in the course of a geological period. The yield stress we invoke in the present hypothesis is less than one-hundredth of the smallest stress used by Griggs.

These few experimental results, even with the theory of Goranson, leave us far short of verifying that continental basement rocks will flow plastically in the conditions postulated, but they do demonstrate the importance of two factors—time,

and the presence of water—and thus give valuable guidance on the direction that further experiments should take. Meanwhile it would appear that the results of experiments in which these factors are not given due weight cannot be accepted, any more than can the existence of mountains, as compelling evidence that the crust has great and permanent strength throughout its thickness.

3. A superficial plastic layer

A geographical property of continents that invites explanation is that the average heights above sea-level, whilst varying over a rather narrow range of values, are closely related to the areas. Estimates of the average heights as compiled by Kossinna (1933) are shown in Table 1 and reveal the small range of variation 0·34–0·96 km, if we may omit Antarctica for the moment on account of its ice-cap. Within this range the average height is seen to increase with the area—from Australia, the smallest and lowest continent, to Asia, the largest and highest.

Table 1

Dimensions of continents

	Area of land (km^2)	Average height (km)
Asia	44 100 000	0·96
Africa	29 800 000	0·75
N. America	24 100 000	0·72
S. America	17 800 000	0·59
Antarctica	14 200 000	2·20
Europe	10 000 000	0·34
Australia	8 900 000	0·34

This increase of height with area is rather what we should expect if the continents were merely elevated portions of a continuous surface. We know, however, that the continental margin denotes a fundamental discontinuity in the crustal layers; hence each continent is to be regarded as a separate entity, and the relation between height and area remains to be explained. Such a relation might conceivably arise from erosional effects, but erosion depends so much on climate, topographic relief, the nature of the surface rocks, and other features in respect of which the continents differ widely from one another, that for any such regularity to result could only be fortuitous. But the data of Table 1 are readily explained on the hypothesis that the continents, in protruding above the level of the ocean floor, behave as plastic bodies which are capable of flowing laterally under their own weight. This mode of behaviour has been investigated in much detail for the case of ice sheets, the theory of which has been developed in a series of recent papers by Nye (1951, 1952, 1957). An important feature of plastic flow in sheet-like bodies is that spreading occurs throughout the thickness of the sheet and not by extrusion at the base.

Plastic behaviour is defined for theoretical purposes in terms of a yield point, i.e. a critical value of shear stress characteristic of the material: it is assumed that no deformation occurs at stresses below the yield point, and that plastic flow at the yield point prevents the development of any greater stress. This idealization has led to useful results in the case of ice, for which the actual relation between stress and rate of strain (Nye 1951) is rather similar to that found by Griggs for

gypsum. The bedrock under an ice sheet is regarded by Nye as a rough motionless plane. No such boundary can be postulated for the continents; we may presume in the first instance, however, that horizontal movement is relatively inhibited below the level of the ocean floor, which therefore constitutes a lower limit for the base of plasticity. The flow mechanism proposed here is thus entirely distinct from any that might be invoked in the deep crust or mantle.

The base of the plastic layer in the continents is likely to occur at a somewhat shallower level than the lower limit just mentioned; in fact there are physical grounds for expecting the plasticity of basement rocks to diminish markedly at depths greater than 2 or 3 km below the surface. From explosion seismology it is well known that the velocity of seismic waves in a particular type of rock increases with depth. The greater part of the increase occurs at shallow levels and is attributed to the effect of pressure in closing the pore spaces in the rock (Gutenberg 1955). The phenomenon has also been studied in the laboratory. Hughes & Cross (1951), for example, determined the velocity in several types of basement rock at pressures and temperatures corresponding to a series of depths in the ground. A graph of their results (Figure 1) shows that in two specimens of granite and one of marble, about four-fifths of the velocity change would be expected to occur in the top $2\frac{1}{2}$ km, while a specimen of quartzite would reach a maximum velocity at about this depth. Similar rapid initial increases of velocity with pressure have been observed in greywacke, schist and gneiss (Birch 1955), and Birch also states that the porosity of igneous rocks can usually be reduced almost to zero by pressures corresponding to about 3 km depth. Now porosity is a necessary condition for the presence of free water. Thus, if solution and recrystallization constitute the mechanism of plastic flow, as we have suggested above, deformation will tend to be inhibited as the porosity is reduced to a low value. Figure 1 and the other results quoted show that the effective base of the plastic zone might be found at a depth of around 2–3 km below the continental surface.

Although the porosity of basement rock appears to diminish steadily with increasing depth, the plasticity may not be much affected until the base level is approached. Goranson (1940) has shown on theoretical grounds that if the solvent is under pressure the solute is more mobile and plastic flow is encouraged. The mobility of the solute also increases with temperature. These effects will tend to offset that of diminishing porosity, but eventually the amount of solvent present will no longer be adequate to support the flow process at all. In the absence of experimental data on the relative significance of these effects, we shall make the simple assumption that the yield point of continental basement rock has a constant finite value in the depth range 0–$2\frac{1}{2}$ km, and is virtually infinite at greater depths. Moreover, a horizontal plane will represent the base of plasticity sufficiently well if we are considering a continent as a whole, since particular irregularities of topography are relatively short-lived.

The pressure at a depth of $2\frac{1}{2}$ km on land is attained in regions of deep ocean at about the surface of the basaltic layer, but the temperature in the latter case is considerably lower. Thus if the relation illustrated in Figure 1 holds for basalt (and Hughes & Cross have shown that it does hold for dunite), we must conclude that there is little or no plastic layer in the ocean deeps. We may therefore assume that the continental plastic layer thins out oceanwards to terminate at the bottom of the continental slope. Figure 2 shows the estimated position of the base of plasticity in relation to the hypsographic curve. From this figure it is apparent that the present hypothesis identifies the continental slope with the steepening

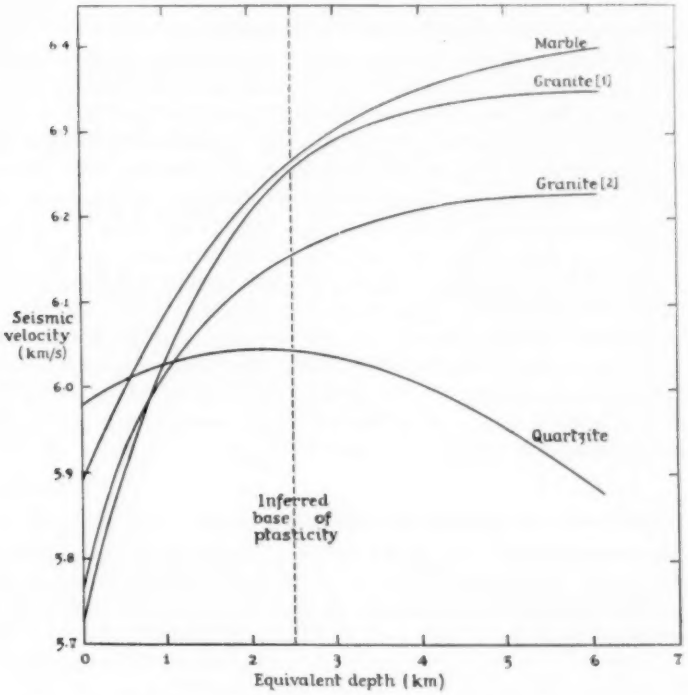

FIG. 1.—Variation of compressional velocity with depth for basement rocks, showing the effect of the closing of pore spaces. From laboratory measurements by Hughes & Cross (1951).

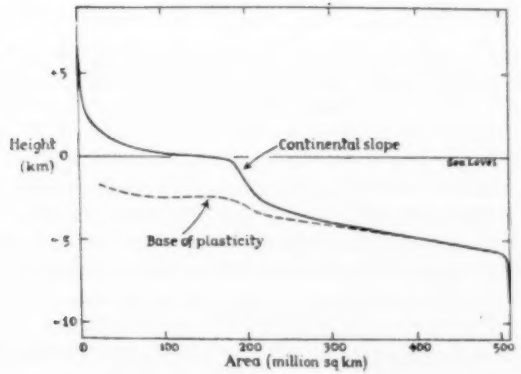

FIG. 2.—The hypsographic curve (after Kossinna 1933) and the estimated position of the base of plasticity.

edge of the plastic slab and thus offers a simple explanation for the existence of this remarkable geomorphological feature. Detailed evidence in support of this suggested origin of the continental slope will be discussed below.

At the surface of the oceanic basement the pressure is already sufficient, as we have seen, to reduce the porosity of the rock to a very small value. Independent evidence that this is so is perhaps to be found in the fact that the seismic *Lg* phase does not propagate in oceanic regions. For Birch (1958) has suggested that the propagation of this phase depends upon precisely that near-surface velocity gradient that we have been discussing, and have illustrated in Figure 1. One may rely as confidently on the successful propagation of *Lg* waves in a continental crust as on their failure to survive in an oceanic crust; in fact this phase provides a criterion for deciding whether the path traversed is wholly continental or not (Oliver, Ewing & Press 1955). Since we have linked the mechanism of plastic flow to the same velocity gradient, we may suppose that the flow phenomenon cannot affect more than a relatively thin layer beneath the deep ocean floor. It is a consequence of this deduction that if the subsidence of guyots is attributed to plastic flow this must occur not in the primaeval oceanic crust but at shallower levels, in the material of the guyots themselves.

4. The continents in plastic equilibrium

The theoretical profile of a plastic slab in equilibrium is approximately parabolic (Nye 1951). An idealized picture of a typical continental slab is shown in Figure 3. Here the part of the surface profile that lies below sea-level is merely

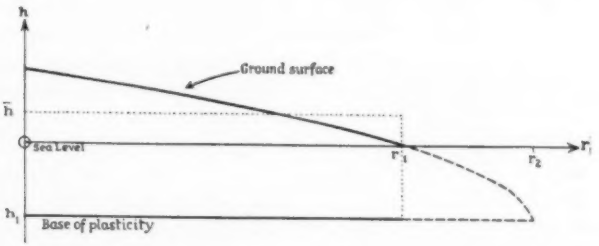

FIG. 3.—Theoretical radial section of continental plastic slab showing idealized ground surface and base of plasticity. Vertical exaggeration: 300 times approx.

an extrapolation of the land profile; the actual submarine profile will be steeper because of the downward trend of the base of plasticity, and the water pressure will tend to maintain this added steepness. For the moment, however, we are concerned only with the land portions of the continents. The equation determining the height h at radius r for the idealized surface in Figure 3 is

$$h = h_1 + \{2h_0(r_2 - r)\}^{1/2}$$

where h_1 is the (negative) height of the base of plasticity, r_2 is the radius of the slab, and

$$0 = k/\rho g$$

where k *is* the yield point and ρ is the density of basement rock, and g is the acceleration of gravity. The value of h_0, which is presumably the same for all continents,

will be derived below. In Figure 3, $\bar{h}$ is the average height of the land above sea-level.

The volume of a circular continental slab out to the coastline ($r = r_1$) is given by

$$\begin{aligned} V_{r_1} &= \pi r_1^2(\bar{h}-h_1) \\ &= \int_0^{r_1} 2\pi r(h-h_1)dr \\ &= (4\pi/15)(2h_0)^{1/2}\{2r_2^{5/2}-(r_2-r_1)^{3/2}(2r_2+3r_1)\} \end{aligned}$$

so that

$$\bar{h} = h_1+(4/15r_1^2)(2h_0)^{1/2}\{2r_2^{5/2}-(r_2-r_1)^{3/2}(2r_2+3r_1)\}.$$

Also when

$$h = 0, r = r_1$$

therefore

$$r_2 = r_1+h_1^2/2h_0.$$

Thus, writing

$$\pi r_1^2 = A,$$

$$\bar{h} = h_1+(4\pi/15A)(2h_0)^{1/2}[2\{(A/\pi)^{1/2}+h_1^2/2h_0\}^{5/2}- -(h_1^2/2h_0)^{3/2}\{5(A/\pi)^{1/2}+h_1^2/h_0\}].$$

This in effect gives us a relation between $\bar{h}$ and A for the continents, with the single disposable parameter h_0; for we shall postulate that the average thickness of the slab is 2·5 km for all continents, i.e.

$$h_1 = \bar{h}-2{\cdot}5.$$

We thus have a theoretical relation between the average height and area of a continent, for comparison with the data already given in Table 1. The comparison is shown in Figure 4, where the theoretical curve has been computed for

$$h_0 = 0{\cdot}00165 \text{ km}.$$

The variation of continental height with area thus appears to be accounted for by regarding the top 2½ km of each continent as a plastic slab. Apart from the case of Antarctica, which will be considered separately below, the degree of fit obtained in Figure 4 is better than one might have expected, for there are several likely sources of scatter. It would be more logical to redefine the continents as including their submarine margins, but accurate estimates of average depth are not yet available for the marginal zones. In some cases the continental boundaries may need further revision. One might ask whether the coalescence of Europe and Asia has not been long enough established to warrant treating them as a single continent. Such a continent of Eurasia has been included in Figure 4 as an alternative. For some continents the assumption of circular shape is rather crude; it can be shown, for example, that the correction required for an elliptical slab, if it is to be compared with circular ones, is a reduction of the area in the ratio of the axes (J. E. Drummond, personal communication). This correction would bring Eurasia close to the theoretical curve in Figure 4; though the curve would no doubt be drawn rather differently if the areas of other continents were corrected for shape.

Differences of topography will also produce some scatter in Figure 4 since the detailed pattern of flow must be affected by major topographic features, but one may note that the volume of rock above sea-level in a continent is only about one-quarter of the whole plastic slab, and further that the height of the most majestic peak is but a few thousandths of a continental diameter. It is nevertheless true that a large fraction of the total volume of a continent above sea-level may be contained in a relatively small area, as is the case with the Andes in S. America. Thus the surface profile may be very different from the parabolic shape that the ideal theory implies. If N. America, for example, conformed to this theoretical

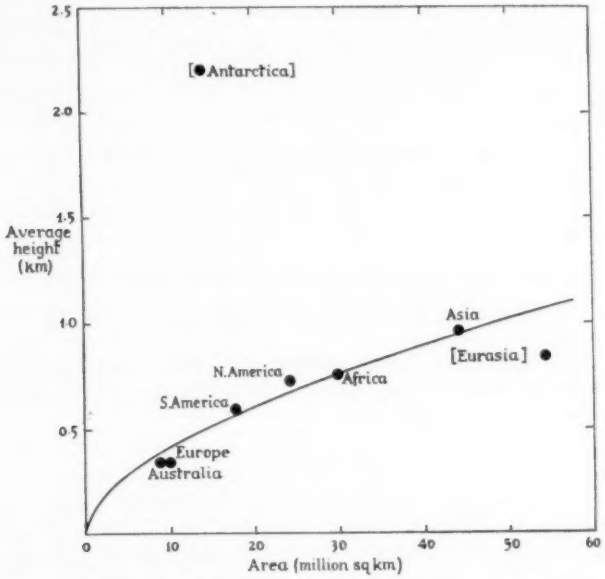

FIG. 4.—Height/area data for the continents, with theoretical curve for circular plastic slabs in equilibrium.

shape we should find that the surface would rise steadily to a height of about 1·8 km above sea-level at the centre. Departures from this configuration may be attributed, however, to erosion and to geotectonic movements. The main effect of erosion is to transport material within the limits of the continental margins, though the area above sea-level is probably enlarged in the process. Mountain ranges and geosynclinal depressions occur side by side (Bucher 1933), and it appears that uplift in one place may be largely offset by sinking in another. Thus, although these influences may greatly alter the ideal parabolic profile, the average height of the continent still approximates to that of a plastic body in equilibrium.

The yield point for continental basement rock is calculated from the value of h_0 by putting $\rho = 2{\cdot}67\,\mathrm{g/cm^3}$ and $g = 980\,\mathrm{cm/s^2}$. Thus $k = \rho g h_0 = 0{\cdot}43 \times 10^6$ dyn/cm^2. This value is about one-thousandth of the strength of crustal rocks against immediate fracture, as calculated by Jeffreys (1952). It is less than one-hundredth of the smallest stress-difference used by Griggs (1940) in his studies of flow in gypsum; this particular experiment was continued for 500 days, but the likelihood has already been mentioned that very much lower yield points may

come into operation on a time scale more appropriate to the context of the development of continents.

Antarctica is the only continent that does not conform moderately well to the trend shown in Figure 4, but here the discrepancy is a violent one: the average height above sea-level is 2·2 km instead of about 0·50 km. In 1927 Meinardus compared the average height of Antarctica with the mean of the other continental regions in the southern hemisphere, and concluded that the difference (1·6 km) represented the thickness of the ice-cap. If we use the more extensive data of Figure 4, and assume complete isostatic adjustment to the load of the ice-cap, we may deduce an average ice thickness of 2·37 km. (Densities have been taken as 0·92 g/cm^3 for ice and 3·27 g/cm^3 for the mantle.) This value would until lately have been regarded as much too great (Sharp 1956), but it is just the sort of value that is suggested by the recent seismic soundings, many of which place the rock surface well below sea-level (Woollard 1958; Wexler 1958). Moreover, we may calculate the sinking due to isostasy as 0·67 km; hence the edge of the continental shelf should lie at a depth of about 0·8 km, and a glance at the bathymetric map shows that this is so. These estimates provide grounds for counting Antarctica in with the other continents as displaying a general condition of plastic equilibrium.

To maintain the average heights of the continents at a level close to that of equilibrium requires some means of raising the surface at a rate not less than that at which it is lowered by the loss of sediments to the oceans. For the source of this supply we may look to volcanism, which has already been invoked by Wilson (1954) in his theory of continental accretion. According to some theories the orogenic cycle yields a net increase of surface elevation, and this is another possible source of supply. We may presume that either mechanism would operate at a fairly constant rate throughout time, and Wilson points out that the present rate of volcanic deposition is sufficient to have supplied the total volume of the continents. This rate has been estimated at 1·0 km^3/yr for the whole Earth, corresponding to a net average thickness of about 10^{-8} km/yr deposited on all present land surfaces. We shall adopt this value below for purposes of calculating the magnitude of certain movements that we shall attribute to plastic flow. Actual rates of height increase will vary from region to region and from period to period, perhaps by a substantial factor. Nevertheless, for each continent we envisage small increments of elevation leading to a slow areal expansion, more or less irregular in space and time, the continental mass as a whole never departing far from the condition of equilibrium. Wilson has stated that "there seems to be a process of growth by which continents expand and encroach upon ocean basins". Here we have seen that plastic flow under gravity provides such a process.

The height of the continents has often been supposed to bear some relation to sea-level (e.g. Kuenen 1955). On the present hypothesis it is related fundamentally to the level of the ocean floor, with the depth of the sea having a secondary effect. Also involved in these relations, according to isostasy, is the thickness of the continental crust. Worzel & Shurbet (1955a) have been led by the best available evidence to define standard, isostatically equivalent sections of continental and oceanic crusts. From their results it follows that actual continental crusts should vary in average thickness from about 35 km for Australia to about 40 km for Asia, following the variation in average height. This narrow range is certainly consistent with present knowledge of crustal thicknesses, though we cannot claim to have measured the average thickness with such precision. Thus the plasticity that determines the height of the continents must also be held responsible, in conjunction with

isostasy, for the characteristic thickness of the continental crust. As we shall see later the flow process, in enlarging a continent, builds *downwards* around the margin until the standard thickness is attained. This concept is in marked contrast to the proposal of Gilluly (1955) that the continental crust is maintained at its proper thickness by a subcrustal flow of former sediments towards the continents.

If the elevations of the continents indicate the equilibrium state for land masses of continental size one should expect smaller areas with comparable elevations to be unstable. We have already discussed the instability of volcanic islands and seamounts. It is perhaps significant that no land masses exist that have an area more than a small fraction of that of Australia and do not form part of a continent, as Greenland does (Oliver, Ewing & Press 1955). New Zealand, with an area only 3 per cent of that of Australia and a much greater average height is one of the largest isolated land masses of less than continental size; yet in Figure 4 it would be approximately represented by a point at 0·5 km on the "height" axis. New Zealand is well known for seismic and volcanic activity, and a large proportion of its area has been affected by the latest orogenic revolutions. Associated with these features one may expect to find a relatively rapid plastic flow, which will tend to increase the area and reduce the elevation until some point on the curve of plastic equilibrium is reached (Figure 4). Thus the fact that there are so few large isolated islands may be a consequence of their instability as plastic bodies; at equilibrium the surface of such a body would be far below sea-level.

5. The inert shields

The continents are made up of two geologically distinct types of region: the continental shields and the orogenic and geosynclinal belts. The shield regions have been comparatively little studied, yet they together constitute some two-thirds of the total area of the continents. The history of the shields is known to extend back for at least 2700×10^6 years, whereas the present regions of orogenic and geosynclinal belts date from no more than about 500×10^6 years ago. The shields have remained almost undisturbed during the lifetime of the folded belts although they bear the evidence of severe deformation in earlier times. This is the main significance of the difference of age between the two types of region; for ancient rocks are also sometimes found incorporated in the strata of young mountains, and on the other hand there are large areas of shield which carry a veneer of younger though virtually undeformed sediments. The shield regions are further characterized by extensive and advanced metamorphism—more so than is usually found in the orogenic regions.

The part that the shields have played in continental development, and especially the relation between shields and folded belts, are questions fraught with much confusion. There is no doubt that the shields are highly stable; yet because the orogenic belts tend to be ranged around the edges of shield areas, and often at distances increasing with the lateness of the orogeny, these inert masses have been called "dynamic centres of tectonic activity" (Umbgrove 1947, p. 30). The term hinterland would seem expressive of the relation in which the shields stand to the surrounding orogenic belts; yet because in some places the high-standing younger formations appear to be riding over the adjacent shield area of lesser elevation, some writers have described the shield as a foreland. In all the hypotheses that attribute geosynclinal and orogenic movements to thermal effects in the mantle one is invited, as van Bemmelen (1954) has pointed out, to picture neighbouring

regions of inert shield as actually in motion like the jaws of a vice, squeezing upwards and downwards the intervening crust and sediments.

The detailed configuration of the crust in shield regions should set some bounds to the scope of speculation regarding their origin. The thickness of the crust under the Canadian, African and Australian shields has been determined seismologically by means of rock bursts or artificial explosions, and values of about 35 km have been found in each case. Within a shield one can discern separate provinces, distinct one from another, both in the degree of antiquity and in the general strike of the rocks. These provinces appear to be the remnants of ancient orogenic belts, and from the highly contorted nature of the formations and also from the notion that long-continued erosion must have removed enormous thicknesses of the original mass, it has come to be believed that what we now see are the very roots of the ancient mountain chains. Such a conclusion seems incompatible with a present crustal thickness of 35 km, which is more or less typical of continental regions as a whole, unless there is widespread replenishment of the crust from below. According to van Bemmelen (1954, p. 24) the roots of mountains have "accumulated from below by natural geochemical processes which are a reaction to a geosynclinal subsidence"; but no such reaction could arise beneath the inert shields, nor could roots formed in this way display the characteristic foliation of the shield gneisses. Nevertheless we may accept that the rocks now exposed or thinly covered by sediments in shield regions were formerly buried at depths of perhaps 10–15 km in ancient orogenic belts. We shall see later that the degree of contortion exhibited by such rocks should increase with the depth to which they were once buried, and while agreeing with Wilson's (1954) suggestion that this complexity of structure may have been acquired in part when the rocks were sedimentary beds slumping on former continental shelves, we shall propose a further source of contortion in the piling up of pre-existing basement rock at the margin of the expanding continent.

How may we include the vast inert shields in our picture of the continents as slabs of plastic material? The most direct clue is given by the height at which the shield regions stand as compared with the height of the continents as a whole, together with the comparative flatness of most of the shield surfaces. The estimates usually quoted for the average height of all the continents and for that of the shield regions (e.g. Poldervaart 1955) are considerably affected by the inclusion of the Antarctic ice-cap. If we modify these estimates by taking the average height of Antarctica as 0·50 km, as suggested above, we arrive at about 0·7 km for the average height of the continents above present sea-level, 0·6 km for that of the shield regions, and 1·0 km for the regions of orogenic and geosynclinal belts.

Let us suppose that the height of a large region extending into the interior of a plastic slab is somehow reduced below the equilibrium value. Then although the rest of the slab is flowing, or begins to flow, the low-lying region will tend to remain motionless. But in a slab that is close to the equilibrium state the occurrence of flow is in general associated with an increasing height; hence any region that ceases to participate in the flow becomes progressively less capable of doing so. Furthermore, if by remaining motionless such a region should be denied the replenishment that is the ultimate cause of the flow phenomenon, then its movement may have ceased for all time. It is along these lines that we may visualize the establishment of the continental shields.

The initial reduction of height in a prospective shield region may occur by a combination of erosion and the lack of activity inherent in the central region of a

plastic slab. The theory of Nye (1951) shows that this central region is always underlain by a mass of stagnant material. The conditions close to the centre of an ideal plastic slab at or near equilibrium are illustrated in Figure 5; the two cycloidal slip-lines which emerge precisely at the centre delineate a buried conical mass of stationary material. So long as the surface continues to be replenished, or at least is not denuded, no appreciable area of the stationary mass will outcrop; but if we imagine a horizontal slice removed from the top of the slab we can see that a stagnant region will be revealed which will tend to increase in area. Now the disturbance constituted by a continually thickening and spreading sheet of rock must in some measure be projected into the substratum, and may conceivably assist the promotion of volcanic and other activity. Such an effect will be a minimum at the centre of the slab because of the presence of the stationary cone; thus the central region will tend to lack replenishment in the form of volcanic deposits, whilst erosion, perhaps aided by isostatic recovery, will proceed to reveal an increasing area of stationary rock. Eventually denudation will expose rocks that were formerly deeper than the base of the plastic slab and have been motionless since an altogether earlier phase of their history.

This theoretical picture of the origin of a shield is over-simplified, since the stationary cone depicted in Figure 5 (in which the vertical and horizontal scales are the same) is a very small body of rock in comparison with existing shields. From

FIG. 5.—Section through centre of a thin plastic slab near equilibrium, showing slip-line field and inert cone (after Nye 1951). Arrows indicate directions of incipient flow.

the cycloidal nature of the slip-lines it readily follows that the base diameter of the cone is barely more than five times the height, e.g. for an ideal plastic continent of the size of N. America the base of the central inert cone would be about 23 km in diameter. But just as each shield incorporates several provinces, so the fine structure of foliation within a province seems to have resulted from the coalescence of a large number of small nuclei. This is strongly suggested by the aerial photograph and structural map of parts of Grenville Province, N. America, reproduced by Wilson (1950). The theory described above may well account for the initial immobilization of each small nucleus, leading eventually to the formation of a large inert shield.

Among the characteristics of shield regions is the low incidence of volcanoes and earthquakes. It has been suggested above that although such a region becomes confirmed in a state of inertia through a lack of volcanic activity, this lack itself results from the prior cessation of flow. Conversely we shall suggest later that the various disturbances which characterize active continental margins may be to some extent promoted by the persistent and vigorous flow there. But let us briefly consider the relations between actual shields and their environs. The simplest case is that of Africa (including Arabia and Madagascar), which is virtually one great shield, and displays little orogenic or seismic activity, with moderate Quaternary volcanism. Now it can be said of Africa more than of any other continent, that the interior is largely made up of highlands; in other words, Africa conforms most closely to our ideal picture of a plastic slab in equilibrium. This circumstance,

which can only be regarded as fortuitous in view of the typically complex structure of the African shield, may be held to explain the stability of the continent as a whole. Evidently erosion has not succeeded in seriously modifying the equilibrium profile of Africa, and it will be seen in what follows that although the low-lying shields found in other continents are themselves stable, instability is apt to set in at the margins of such shields. In Australia too the ideal configuration is roughly attained, though to a lesser extent than in Africa, and except for the mountainous and disturbed outlier of New Guinea, Australia has been highly stable since the Palaeozoic.

The Antarctic continent is exceptional from the viewpoint of general tectonics in being virtually aseismic (Gutenberg & Richter 1949), in spite of the extensive mountain-building and block-faulting that has occurred since Tertiary times. This peculiarity is perhaps related to the presence of the ice-cap. The ice imposes lateral constraint on the more elevated rock masses directly, and on the continental slab as a whole by reducing its average elevation to below sea-level; moreover, the pressure in the basement rock is increased and the temperature lowered. Each of these effects tends to inhibit plastic flow in the rock, and it seems possible that earthquakes have been temporarily suppressed as a consequence. One might say that with respect to dynamic phenomena generally the Antarctic continent has been placed "in cold storage".

The other continents present an entirely different picture. In each case the shield region falls far short of providing the high-standing interior that stability demands, and the orogenic belts are a continuing source of disturbance projected back upon the neighbouring areas of shield, not only in the form of erosional material, but by gravitational gliding and plastic flow. It is estimated that the amount of sediment that has been deposited on areas of shield would be sufficient to cover all shield regions with $\frac{1}{2}$ km of solid rock (Poldervaart 1955). The so-called "overthrusting" of classical geology, by which shields are typically encroached upon by the younger mountain chains along their borders, may well be mainly a result of plastic flow under gravity. Referring to the thin but extensive "thrust sheets" that characterize the eastern front of the Rocky Mountains, sometimes encroaching upon the adjacent shield to distances of several hundred miles, de Sitter (1956) voices the suspicion that many such features may be partly due to gliding. From his description they seem to be very much as one would expect as a result of plastic spreading of the base of a mountain range.

On the classical view it is permissible to describe any example of this type of phenomenon in terms of either overthrusting or underthrusting. Often these expressions are used to denote mere spatial relationship, however, and it would appear that the original concept of thrust, implying an external horizontal force, has contributed little to an understanding of the field observations. With the present hypothesis we visualize part of a high-standing formation flowing over on to the shield, and if we were to insist that the shield moves in, relatively speaking, under the younger formation, we should be losing sight of the causative process altogether. For the same reason one may find difficulty in agreeing with the remark of de Sitter (1956, p. 193) that "even in structures due to gravity gliding it is immaterial whether we think of a sedimentary cover gliding down or a rising block". It should be mentioned that the process of plastic flow under gravity has nothing in common with the fluid pressure mechanism recently proposed by Hubbert & Rubey (1959) to account for "overthrusts", except that the presence of water in the pores of the rock is required in both cases.

The role of the low-lying shields is thus by no means that of "dynamic centres", but of passive receptacles for the erosional materials and the sheets of flowing or gliding rock cast off by the neighbouring orogenic belts. The shields encourage this overriding process, not primarily by virtue of a superior rigidity, as thrust tectonics would have it, but because they are low. Only when the additional load is sufficient to upset the isostatic equilibrium of the shield area does it become the site of deeper disturbances. Yet for some reason the shield may be disturbed earlier and more violently than the continental shelf region on the other side of the orogenic belt, or so it would appear from the case of the N. American Cordillera. Here, and even more in Eurasia, there seems to be a strong tendency for orogenic activity to rebuild the continental interior, and thus to restore the surface profile to a condition more nearly resembling that of plastic equilibrium.

6. Block faulting

A region of continental shield where the elevation has not been reduced much below the equilibrium value may be induced to start flowing again by various means; for example if the surface is raised once more as a result of volcanic or orogenic activity, or even perhaps if sea-level should fall by a substantial amount. Such a revival cannot often occur, for we have seen that most shield regions are distinctly low-lying. Major tectonic movements within shield regions lead typically to the formation of faulted blocks and rift valleys. These spectacular features are interpreted here as resulting from renewed plastic flow in regions of long-established stability.

Two main theories have been propounded to explain the formation of rift valleys; both rely on a general state of stress extending to the bottom of the crust, but in one theory the stress is compressive and in the other tensile. Normal block-faulting also is sometimes ascribed to crustal compression but more usually to tension. Here, however, embarrassment also arises, for some of the most extensive block-faulted belts are flanked by regions where the crustal stress is supposed to be compressive. King (1955) discusses the block-faulted Basin and Range province of the N. American Cordillera, and having mentioned that such faulting is generally consequent upon the adjacent orogenic movements he continues as follows: "Such relations are frequently ascribed to a 'relaxation' of the earlier compressive forces, but as knowledge of the fracture belts increases it becomes difficult to separate in time movements that appear to be primarily tensional from movements that are primarily compressional". We have seen above that such apparently compressional structures as "overthrusts" may sometimes be no more than overflows. We shall now suggest that normal block-faulting does not require a general state of stress in the crust but may result from the plastic flow of a surface layer 2 or 3 km thick, the only applied force being that of gravity. The same process in different circumstances may produce what is called "thrust-faulting".

The pattern of flow in an ideal plastic slab near the equilibrium state is shown in Figure 6a. The slip-line field, which is depicted for the case of movement from left to right, indicates the trajectories of maximum shear stress. Thus if the material towards the right side is caused to flow at too great a velocity for that at the left side, fracture will occur by shearing along one of the family of slip-lines which leaves the surface at an angle of 45° and meets the base tangentially (Nye 1951). A single small normal fault originating in this way is depicted in Figure 6b. Other faults that may develop will also tend to follow slip-lines, either of the same

family or of the orthogonal one. The actual position of the fractures will be influenced by any irregularity in the properties of the slab. If the fractures all follow the original family of slip-lines the result may be somewhat as shown in Figure 6c; if subsequent fractures follow the other family we may arrive at the situation of Figure 6d. In general the detached blocks will become tilted because of the curved shape of the slip-lines. The length of the original slip-lines is about 3 times the thickness of the slab, i.e. for continental slabs some 6–9 km, but a fracture exposed throughout this length would have zero dip at the bottom.

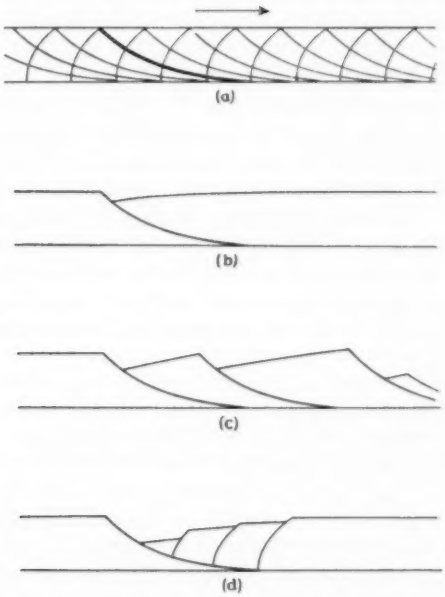

FIG. 6.—Section through plastic slab in extending flow, showing development of normal block-faulting (c), and antithetic faulting (d), by shear fracture along slip-lines. Horizontal and vertical scales are equal.

The usual idealized diagram of normal block-faulting closely resembles Figure 6c, whilst Figure 6d might well serve to illustrate the case of a normal master-fault with subsequent antithetic faulting. These representations do not of course include the effects of variable natural conditions such as the grain of the rock. The following salient features of normal block-faulting are quoted from de Sitter (1956): "A normal fault can have a slip from a few centimetres up to some 3 000 m. There is some evidence that the dip near the surface may be greater than at greater depth. In general it varies from 45°–70°, but in some instances dips of even less than 45° have been ascertained. . . . The nature of the rocks also influences the dip. . . . The width of the blocks between the steps varies greatly, and may be anything from a few metres up to scores of kilometres. There is no apparent correlation between the amount of slip and the width of the steps or any other feature. Both tilting of the steps and antithetic faulting are probably due to the flattening of the fault-face downwards. . . . Individual normal faults have the same characteristics, whatever their cause. . . ." It will be seen that these features are readily explicable in terms of plastic flow in a slab of rock some 2–3 km thick. The revival of flow in

those shield regions where normal block-faulting has occurred may be attributed to an outward movement of the adjacent orogenic belts towards regions of relatively low elevation. In the case of the Basin and Range province the main cause must be sought in a general movement of the Sierra Nevada and Coast Ranges towards the Pacific Ocean.

The conjugate phenomenon of "thrust-faulting" may also be explained in terms of the flow of a plastic slab, as illustrated in Figure 7. The difference in this case is that the material towards the left side is caused to flow at too great a velocity for that on the right side; the slip-lines along which shear fracture tends to develop now rise towards the surface in the direction of flow, emerging at 45°. Again the angles will in practice be influenced by the grain of the rock. The surface manifestation will usually be complicated by the effects of secondary fracture, which will leave the appearance of normal rather than reverse faulting (Figure 7c). As with normal block-faulting, steep "thrust-faults" or "upthrusts" are a characteristic of shield regions; they are particularly common at the margins of mountain ranges where the outward flowing elevated region is confronted by a comparatively stable lowland. De Sitter (1956) has described a succession of "upthrusts" on the southern flanks of the Lombardy Alps. In this region there are also many examples of nappe formation, and it is easy to understand the intimate relationship that is apparent between the nappes and the "upthrusts" when both are regarded as manifestations of the same process, i.e. plastic flow under gravity.

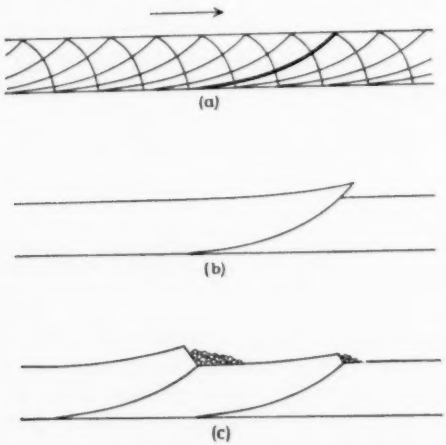

FIG. 7.—Section through plastic slab in compressive flow, showing development of reverse faulting ("upthrusts") by shear fracture along slip-lines. Horizontal and vertical scales are equal.

The two distinct types of flow that have been invoked above to account for normal faulting on one hand and "thrust faulting" on the other are well known both in glaciology and in soil mechanics. Nye (1957) has analysed these modes of flow in connexion with glacier movement and has designated them as "extending flow" and "compressive flow" respectively; in soil mechanics they are referred to as the active and passive Rankine states. Adopting Nye's terminology, we may speak of large-scale normal faults as extensional faults and "upthrusts" as compressional faults. These terms are strongly reminiscent of the classical interpretation of

major faulting as due to crustal tension or compression, but in fact the interpretation suggested here bears only a superficial resemblance to the classical one. As a basic cause we do not have to postulate forces exerted upon the crust from the Earth's interior, but only the effect of gravity on a plastic layer at the surface. One consequence of this distinction is that the occurrence of tensile and compressive phenomena in proximity is no longer an embarrassment, but is rather to be expected, just as regions of extending flow alternate with regions of compressive flow as one travels down a glacier (Nye 1951, Figure 4).

Normal block-faulting exhibits many features that are consonant with the hypothesis of flow, as has been shown above. A recent study of one of the best known block-faulted regions—the Basin and Range province—has included two kinds of measurement that are very much to our present purpose (Thompson 1959). These concern the value and variation of gravity across the region, and secondly the rate of horizontal strain. Perhaps the most important conclusion drawn by Thompson from his gravity traverse, which extended across six ranges and as many basins or valleys, is that there is no perceptible relief at the bottom of the crust to correspond with these substantial features at the surface; i.e. the surface irregularities are uncompensated. With the ranges only some 35 km apart this is rather what one would expect if the depth of the faulted blocks is limited to 2–3 km. Thompson actually postulates faulting to the bottom of the crust, but in such a way as to produce no vertical movements there; to achieve this he adopts the novel and unverifiable assumptions that the lower part of the crust has been extended plastically or dilated by igneous intrusions. Nor is it clear how tilting is to be brought about in Thompson's model.

Another highly significant result from the gravity measurements is that there is an average isostatic anomaly of about −20 mgal over the entire region surveyed. Thompson mentions that "over-compensation sufficient to support topography a few hundred feet higher throughout the region" would give an anomaly of this magnitude. In his later discussion of the observed horizontal strain in this region Thompson estimates that the total extension of the surface in the general direction in which the faults dip, i.e. approximately E–W, amounts to 5 per cent. If this extension has not affected the crust throughout its thickness but is confined to a surface layer 2–3 km thick, it follows that the layer is now an average of 5 per cent or 100–150 m thinner than it was to begin with. Thus the apparent over-compensation suggested by Thompson is very simply accounted for, though one is left with the question as to why there has been no regional isostatic uplift to make good the deficiency.

In a circular plastic slab the theoretical rate of strain in the radial direction at radius r is given by $\partial v/\partial r$, where v is the radial velocity; the variation of v with depth in the slab is negligible (Nye 1951). We may assume that the state of the slab as a whole never departs far from equilibrium. Then

$$v = (\pi r^2\alpha - \partial V_r/\partial t)/2\pi r(h-h_1)$$

where V_r is the volume of the slab within radius r, and α is the average thickness of material effectively added to the surface in unit time. Now we have already seen that

$$V_r = (4\pi/15)(2h_0)^{1/2}\{2r_2^{5/2} - (r_2-r)^{3/2}(2r_2+3r)\}.$$

Hence if V_2 is the total volume of the slab

$$V_2 = (8\pi/15)(2h_0)^{1/2}r_2^{5/2}$$

also

$$\partial V_2/\partial t = \pi r_2^2 \alpha.$$

Thus, writing

$$\partial V_r/\partial t = (\partial V_r/\partial r_2)(\partial V_2/\partial r_2)^{-1}(\partial V_2/\partial t)$$

we find

$$v = (\alpha/4)(2h_0)^{-1/2} r^{-1}\{r_2^{1/2}(2r_2+r) - 2(r_2-r)^{1/2}(r_2+r)\}$$

and

$$\partial v/\partial r = (\alpha/4)(2h_0)^{-1/2} r^{-2}\{(r_2-r)^{-1/2}(2r_2^2 - r_2 r + r^2) - 2r_2^{3/2}\}$$

where, from Figure 3,

$$r_2 = r_1 + h_1^2/2h_0.$$

The E–W component of the radial rate of strain to be expected in the Basin and Range province is given approximately by $(1/\sqrt{2})\partial v/\partial r$, where if we take the city of Winnipeg as the centre of N. America, $r = 1\,900$ km. Let us assume tentatively that the surface of N. America is being replenished at the normal rate, i.e. $\alpha = 10^{-8}$ km/yr. Also $h_0 = 0{\cdot}00165$ km, and for N. America $r_2 = 3\,730$ km. Hence we obtain

$$(1/\sqrt{2})(\partial v/\partial r) = 1{\cdot}0 \times 10^{-9} \text{ per year.}$$

According to Thompson the block-faulting is a product of the last 15 million years, in which time the theoretical strain amounts to 1·5 per cent. This agrees as closely as could be expected with the estimate of 5 per cent which Thompson derived from observational data. The discrepancy would be accounted for if the average rate of height increase in the western U.S.A. since the Miocene were about three times greater than the value that we have taken as normal for all continental regions; this seems reasonable since the western margin is active and the eastern margin is inactive. From the evidence of extension Thompson concludes that "the often expressed idea that basin–range structure is the superficial result of horizontal compression in the deeper crust can hardly be correct". We have seen above that the observed extension is quite well accounted for by the hypothesis of plastic flow in the uppermost 2–3 km. If this is indeed the correct explanation, the block-faulting that characterizes the Basin and Range province is intimately related to the growth of the N. American continent.

7. Rift valleys

A rift valley or something closely resembling it will be the final outcome of normal faulting if movement should continue along a single major fault. This process is depicted in Figure 8. If no subsidiary fracturing were to occur the valley would ideally acquire a shape as shown to scale in Figure 8c, i.e. 2–3 km deep and at least 11–16 km wide. In practice fracturing will occur, especially on the right-hand side where substantial scarps may result, and this together with erosion will make the valley both wider and shallower. Rift valleys are characteristically 1–2 km deep and 30–50 km wide; such dimensions may readily be produced in the manner suggested here (Figure 8d). But if the movement should cease at an earlier stage we should be left with a rift of the same character though on a smaller scale (Figure 8b). The occurrence of such minor rift valleys, for example in the lower

Rhine embayment, is more difficult to explain in terms of fractures that are supposed to extend to the bottom of the crust. Again, the well-known tendency of rift valleys to be asymmetric in section is accounted for by the essentially asymmetric shape of the ideal valley in Figure 8c.

The system of rift valleys in East Africa may be attributed to a revival of plastic flow promoted by early outpourings of the volcanic lava that is so widespread in this region. The relation between volcanism and the rifts has always been obscure. Although volcanic activity has occurred in the floor of many rifts, yet some of the largest rift valleys, such as Lake Tanganyika, seem to be devoid of volcanism. Volcanic activity has been in progress in the rift region since the late

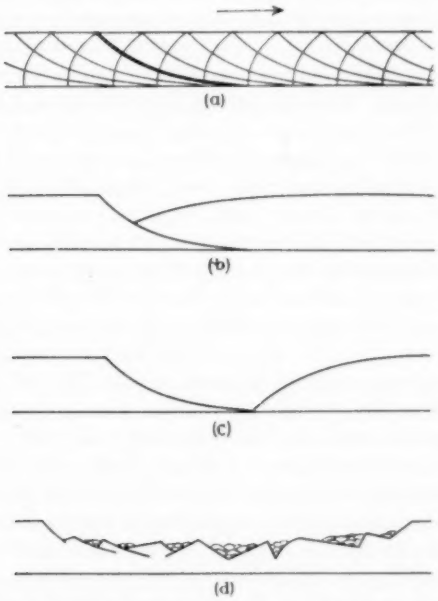

FIG. 8.—Section through plastic slab in extending flow, showing development of rift valley. The ideal shape (c) is made wider and shallower (d) by subsidiary faulting. Horizontal and vertical scales are equal.

Palaeozoic, and the opening up of the rifts is believed to have commenced in the early Mesozoic. Some rifts have cut through pre-existing volcanic deposits, while on the other hand major volcanoes such as Kilimanjaro and Kenya are well removed from the nearest rift valleys. These apparently complex relationships are immediately clarified if we accept that volcanic deposits may give rise to the formation of a rift valley some distance away, and secondly that a rift valley once formed offers only slightly more favourable conditions for subsequent volcanic activity. The hypothesis of plastic flow involves both these propositions, for the movement that produces the rift is widespread—it is that by which continents expand—yet the main disturbance penetrates to rather less than one-tenth of the thickness of the crust.

The actual location of a rift valley may be determined either by a zone of relative weakness in the shield or by a zone of superior elevation. In regions of irregular topography the latter is likely to be the determining factor, and since an elevated

mass is more apt to participate in flow under gravity than is more low-lying ground, one would often expect to find a rift developing immediately to the rear of a line of mountains. This is all the more likely if, as seems probable, the base level of plasticity varies to some extent in sympathy with the surface contour. In these conditions the rift will tend to become fully developed, since the mountains may be said to be "sliding down the slip-lines". We thus see how the same process may produce block-faulting in one shield area and rift valleys in another. It is known that the Basin and Range province had been reduced to a peneplain prior to the era of block-faulting. We can only surmise that in the early Mesozoic East Africa was a region of considerable topographic relief.

The spectacular nature of the rifts is often much enhanced by the presence of elevated belts along their margins. In the theories that attribute rifts to crustal compression or tension the raising of the margins is accounted for as an isostatic effect, and we shall see that this effect would operate even more strongly in terms of the present hypothesis. The topographic map reveals much irregularity in the occurrence of these elevations, however, even along a particular valley margin, and it may be that some of the ranges of mountains that played a part in locating the rifts are still partially preserved along their margins, though sufficient time has elapsed for them to have been largely obliterated. The highest non-volcanic mountain in Africa, Ruwenzori (5 119 m), is flanked on both sides by rifts. The present hypothesis might picture an even more exalted Ruwenzori determining the rear edge of a flowing slab, moving off to leave a rift behind, subsequently becoming anchored, and thus allowing the second rift to develop.

The variation of gravity in the region of the East African rift valleys has been discussed in a well-known paper by Bullard (1936). The remarkable demonstration that several prominent rifts are more or less closely associated with negative swings of the Bouguer anomaly, and consequently with pronounced negative isostatic anomalies, is a result to which any theory of rift valley formation must give due attention. Bullard himself considered that the gravity pattern is qualitatively consistent with the theory of crustal compression; Vening Meinesz (Heiskanen & Vening Meinesz 1958) on the other hand has shown how a tensional origin of the rifts may in principle be reconciled with the gravity anomalies. Neither interpretation accounts for the magnitude of the more pronounced anomalies. Indeed, it seems possible that too much emphasis has been placed upon the more extreme examples reported by Bullard; most subsequent authors have entirely overlooked those cases where the gravity anomaly across a rift was small or non-existent, although these are described with equal care by Bullard. From all the gravity evidence (and Bullard's paper is based on a total of only 87 observations) one is perhaps entitled to conclude no more than that negative anomalies are a likely though not necessary concomitant of rift valleys, and that where the anomaly does occur its position and trend may be related in widely varying degrees to the valley profile.

On the present hypothesis the negative Bouguer anomaly often found across rift valleys is attributed to the roots of former mountains (and partly, as in other theories, to light sediments lying on the valley floor). These are the mountains which determined the location of the rifts, in the manner described above, and were carried off their roots by the general flow movement. Evidence that even a great mountain range may be thus translated is found in Otago, New Zealand, where the Bouguer anomaly (-80 mgal) associated with the roots of the Southern Alps is displaced some tens of kilometres to the east of the present axis of the

range (Robertson & Reilly 1958). As has already been mentioned, many of the displaced mountains in East Africa will presumably have been obliterated long ago. A belt of unexplained negative anomaly comparable in magnitude to those found over the rifts was located by Bullard some 300 km south of Lake Victoria; this anomaly may well be the only remaining surface evidence of an ancient mountain range, the roots of which are still preserved beneath the crust. The limited number of observations on the anomaly suggest that it may not have been suitably placed for rift valley formation, considering that the general flow movement in East Africa must have been directed towards the Indian Ocean.

But we have seen that the presence of mountains is not essential to rift formation. Bullard's observations on the Lake Tanganyika rift show a very mild gravity gradient in the north as compared with the south. He also discusses the absence of any significant anomaly in the Naivasha section of the Eastern Rift, although the adjacent sections to the north and south display marked anomalies. Bullard suggests that an anomaly in the Naivasha section may have been cancelled out by flows of heavy lava; on the present hypothesis the position of the rift in this section would be regarded as determined by those of the adjacent rifts rather than by any local irregularity, despite the fact that the region around Naivasha is today marked by considerable topographic relief. Similarly Lake Tanganyika continues along the general trend of the Western Rift even though its northern part was not defined by any very marked anomaly-producing feature. Finally, we may note the absence of gravity deficiency in the rift to the east of Ruwenzori; this would follow from the above suggestion that Ruwenzori became anchored after taking part in the formation of the rift on its western side.

A further aspect of Bullard's gravity observations seems to support the mechanism of rift formation suggested here. This is that "gravity in East Africa is, on the average, slightly deficient". Calculation of a weighted mean isostatic anomaly, based on the 1930 International Formula, revealed a value of about -30 mgal. On the compression theory of Bullard or the tension theory of Vening Meinesz each rift, considered in conjunction with its elevated margins, is in isostatic equilibrium as a whole. Thus these theories do not account for an overall gravity deficiency. But if the rifts are voids left by material that has flowed away horizontally, a net deficiency of gravity in the region must accompany this loss of material and could be made good only by an inflow of subcrustal material over a wide area. It will be noted that the amount of the gravity deficiency is not very different from that discussed previously for the Basin and Range province. The region of the East African rift valleys is comparable in size to the Basin and Range province, and as might be expected from the similar overall deficiencies of gravity it turns out that the amount of extension that has taken place is also roughly the same. And although the evidence of extension in one case is a dual system of rifts and in the other a multiplicity of normal faults, we attribute both to the same mechanism of plastic flow. The fact that the rift valleys of East Africa have a history ten times longer than that of the faulted blocks of the Great Basin is a measure of the contrast between a continent that has been exceptionally stable since the pre-Cambrian and one that appears to be growing at about the normal rate.

8. The continental margin

The expansion of continents by plastic flow should be most clearly evident at the continental margins, where the velocity of flow is greatest, and where the crust

changes from the oceanic type to the continental type in a distance of a few hundred kilometres. Indeed, the geological conditions found at certain margins have already led certain writers to hint at some such process of expansion. Thus Paige (1955) advocates "creep of high-standing continents toward ocean basins", and Gutenberg (1951) draws attention to "some indications that the higher continental layers are spreading slowly over the Pacific basin". The encroachment of continent upon ocean is seen by Stille (1955), however, as an example of thrust tectonics. Thus he speaks of "overthrusting of the broad Pacific block by the surrounding continental blocks". But here too we encounter the ambiguity that seems to be a regular feature of this approach. Stille asks "should we now speak of overthrusting (of the Pacific block) rather than of underthrusting (of the peri-Pacific block)?"; and continues "in any event, both modes of expression are meaningful only in a relative sense". On the present hypothesis we must say unequivocally that the continents are flowing out upon the ocean floor.

The simple theory of the uniform symmetrical plastic slab does not allow for the complexities of actual continents, as we have noted at several points above. One continent may remain undisturbed over a long period whilst others are flowing apace. Further, it seems quite usual for the flow of a particular continent at any given time to occur rather rapidly in a limited sector, and slowly or not at all elsewhere. Thus we have active margins and inactive margins. In N. America, where the margins have been studied more extensively than in any other continent, the Pacific margin is at present active, and the Atlantic margin mainly inactive; there may have been times in the Palaeozoic when the situation was reversed.

A study of the transition from continental to oceanic crust along the northeastern coast of the U.S.A. has been made by Worzel & Shurbet (1955b) using gravity and seismic data. One of their sections is shown in Figure 9. Here the continental shelf is about 140 km wide and a thick layer of sediments extends out across the continental slope and rise, and into the deep ocean. The data were not sufficient to define the layering of the basement rock, but from the shape of the basement section as a whole one can visualize that the oceanward flow of a thin layer at the surface of a continent, together with isostatic sinking, would cause the continental margin to move slowly oceanwards. This proposal gains force from the fact that the seismic velocity towards the bottom of the continental basement, as determined in many regions of the Earth, is similar to that in the oceanic basement (Gutenberg 1955). The view that the so-called basaltic layer of the oceanic crust is continuous across the continental margin, and forms the lower layer of the continental basement, has been put forward previously by Wilson (1954) and others.

At an active margin the layer of sediments shown in Figure 9 would not have had time to accumulate. In discussing their results, of which Figure 9 is an example, Worzel & Shurbet (1955b) make the following comment: "If all the sediment were removed and these areas remained closely in isostatic balance, as they are at present, there would be a continental shelf floored with basement rocks, a continental slope floored with basement rocks, no continental rise, and a deep oceanic area floored by nearly level basement rocks. This must represent an earlier stage in the development of this coast." On the present hypothesis we may place this earlier stage, in the areas studied by Worzel and Shurbet, as contemporaneous with the Appalachian orogenies, when the eastern margin of the U.S.A. was active.

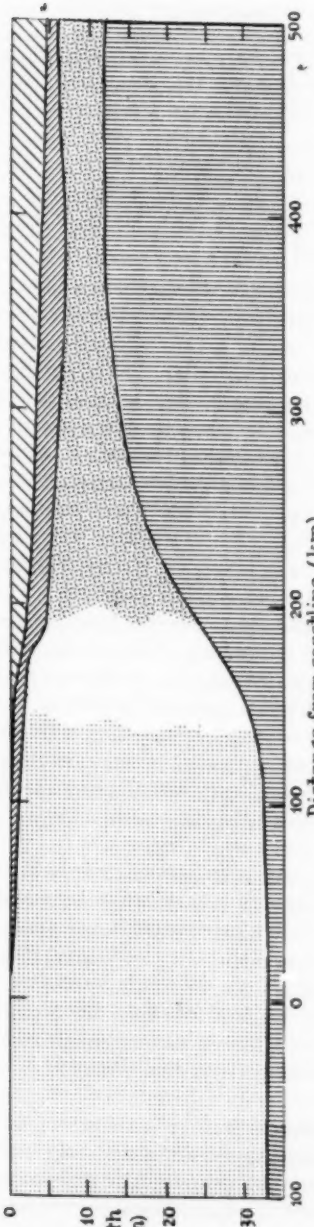

FIG. 9.—Section across inactive continental margin (off Woods Hole, Mass., U.S.A.), as deduced by Worzel & Shurbet (1955b) from seismic and gravity data.

The origin and history of continental shelves is discussed by Shepard (1948), who lists a number of principal facts requiring explanation. The first four of these, relating to shelves in general, are as follows:

"1. The shelves, in general, do not have smoothly sloping, graded surfaces, but are terraced, irregular, and extremely variable from place to place.

"2. The sediments of the shelves do not show a progressive decrease in grade size going out from the shore.

"3. Vast areas of the shelf have rock or gravel bottom and such surfaces are as common along the outer margin as in any other portion.

"4. The sediments of the shelves are, on the average, much coarser than are the sediments now being contributed."

These four important characteristics are all readily explained if we regard the present shelf as a zone that was above sea-level when the continent was smaller and has flowed outwards in the course of continental expansion. Several other features of the shelf appear to support this view. The general gradient near the edge of a plastic slab will be steeper when the flow is rapid than when it is slow; according to Shepard (1948, p. 144), "a striking difference in width is found between shelves located off coastal lowlands and those off mountain ranges, the former being wide and the latter being narrow, particulary off young mountains". The age of present shelf sediments has posed what Kuenen (1950, p. 168) calls "a baffling problem". Kuenen states: "As all observations so far tend to show that the present continental terraces are comparatively young, Mesozoic or Tertiary, one may well ask where are the accumulations of sediment . . . that must have developed during preceding geological ages." It has been mentioned above that the outward flow of a plastic layer at the surface of the continental basement must be accompanied by isostatic sinking at the margin; by this means the basement at each point eventually attains the proper continental thickness. In this process of down-building, successive continental shelves, together with their sedimentary deposits, will be overwhelmed and incorporated in the continental crust. If the oldest existing shelf sediments are Mesozoic it would seem to follow that no part of the present continental margin has been permanently inactive since very much before the Mesozoic.

Again, the absence of pelagic sediments in geosynclinal rocks, to which Gilluly (1955) refers, is easy to understand when we realize that the surface of the oceanic crust, together with its overlying sedimentary beds, must eventually be located some 20–25 km deep in the continental crust. In some regions this buried surface may provide that somewhat elusive seismic interface known as the Conrad discontinuity. The author is indebted to Mr G. A. Eiby for the further suggestion that the buried deposits of oceanic and marginal sediments may form layers of low seismic velocity, such as Gutenberg (1955) has postulated in the continental crust.

The solution of a further difficulty in geology is suggested by the fact, quoted above from Shepard, that shelf sediments do not grade progressively oceanwards as was formerly believed. We regard the unsystematic distribution of shelf sediments as attributable to the horizontal transport of many of them, after deposition, by plastic flow of the basement. It also seems possible that the graded sedimentary sequences that have been adduced as evidence for the former existence of continental masses in regions now oceanic (e.g. Scandia, Appalachia, Cascadia) may likewise have been transported after deposition. Although the rocks that provided

these sediments must at that time have formed an elevated region between the areas of deposition and the ocean, their uneroded remnant may now lie buried beneath the continental shelf and coastal plains, whilst the sediments have been carried undisturbed from inland localities to the vicinity of the present coastline.

The continental slope we have already interpreted as the steep outer edge of the plastic slab. Here again the major features of the margin are related to features of the adjacent land, much in the manner that the flow hypothesis would suggest. Most of the steepest slopes occur off coasts where there is little or no shelf, and the next steepest group occurs off coasts with young mountain ranges. Shepard (1948) estimates that the median angle of the continental slope is 5° 20′ in the Pacific, 3° 05′ in the Atlantic, and 2° 55′ in the Indian Ocean. From this it might appear as if the character of the continental slope were determined by the ocean that it borders, but Shepard himself considers that "such differences as exist between the slopes of the different oceans appear to be more related to the coastal types than to the particular ocean basin".

There is no generally accepted explanation of the origin of the continental slope. The former belief that the slope represented the angle of repose of a large sedimentary deposit became untenable when rock bottom was identified in many places on the shelf and slope; though it is of course true that limited areas of margin off large rivers are largely made up of sediments, and that sedimentation contributes something to most margins. One current theory attributes the slope to a general downwarping of the edges of continents. This view is related to the idea mentioned above that large areas of continent have foundered and now lie beneath the ocean floor. There seems to be little more geological evidence of such downwarping than there is geophysical evidence of submarine continents. Another theory regards the continental slope as a product of faulting. The chief evidence stated by Shepard (1948, pp. 193–4) in favour of this explanation is the frequent occurrence of trenches at the foot of the slope, similar to those found at the base of many fault scarps on land; the seismic activity in the vicinity of many continental slopes; the fact that some slopes cut transversely across the strike of the formations on the shore; and finally the rocky outcrops. To account for this last feature, Shepard invokes recurrent step-faulting, as would also be necessary to produce the typical slope angle of only a few degrees. According to Shepard "it is most difficult to explain the presence of these rocks under other hypotheses".

These features of the continental slope appear to be well accounted for if the continental margin is regarded as the peripheral zone of a plastic slab. The elastic downwarping of the oceanic crust may well begin somewhat beyond the edge of the advancing continental slab, and some extensional thinning of the oceanic crust may also occur; thus trench-like depressions would be formed. Further, the disturbance that is involved in the transformation of oceanic into continental crust may well be accompanied by seismic activity. Mostly, the continental slope will occur in the direction of flow, which is not necessarily normal to the strike of existing land features. Finally, rocky outcrops on shelf and slope are naturally to be expected; in fact one must look for a general topographic similarity between the submarine continental margin and the adjacent land, although along inactive margins the basement relief is largely concealed by sediments. A striking example of this similarity has been revealed in southern California, where the continental slope is separated from the edge of the shelf by a zone more than 200 km wide, of the type usually termed a continental borderland. In this zone there occurs a succession of ridges and basins very similar in topographic relief to the ranges

and basins on shore, and made of the same rocks (Shepard 1948). We may picture these adjacent submarine and land regions as portions of the same plastic slab and subject to the same kinds of deformation; or in some cases it may well be that a topography that has developed on land is carried bodily outwards and becomes partially or wholly submerged.

The seaward translation of landforms as a corollary of the expansion of continents by plastic flow suggests a possible solution of the dilemma that is presented by the submarine canyons. These remarkable features of the continental margin bear a high degree of resemblance to land canyons, as Shepard (1948) has demonstrated in great detail; but they extend in typical cases from shallow water near the shore, across the shelf, to depths of some 2 km on the continental slope. Thus whereas submarine canyons have the appearance of having been formed by subaerial erosion, the possibility can hardly be entertained that sea-level has been 2 km lower than at the present time. The view is now widely held that the erosional agency is provided by turbidity currents, but some writers find difficulty in accepting that these intermittent currents could achieve the necessary amount of erosion, or that the result could be so similar to that of subaerial erosion. The time available for forming the canyons is limited, for although many of them cut into continental basement rock, dredging has shown that the walls are commonly of rock of Tertiary age, including Pliocene.

We may calculate the rate at which a point near the coast is carried seaward, on the present hypothesis, for normal conditions of continental growth. As has already been shown, the radial velocity at the coast ($r = r_1$) is given by

$$v = (\alpha/4)(2h_0)^{-1/2} r_1^{-1} \{r_2^{1/2}(2r_2 + r_1) - 2(r_2 - r_1)^{1/2}(r_2 + r_1)\}.$$

Taking N. America again as a continent of average size, and adopting the same values of the parameters as before we find

$$v = 3{\cdot}5 \times 10^{-6}\,\text{km/yr}.$$

Thus in 10 million years a point near the coast would ordinarily be carried 35 km seawards. This is not quite sufficient to account for the longer canyons, some of which extend as far as 200 km out to sea; moreover, the available time may be even less than 10 million years. But we have already suggested that the rate of height increase α may locally reach values several times greater than our postulated normal value. If we adopt here the value of α indicated above in our discussion of fault movements in the Great Basin of N. America, we arrive at the figure 120 km for the seaward movement of a point on the west coast of the U.S.A. in 10 million years. This is adequate to account for the Monterey Canyon, the largest of the canyons off California. Further, it is generally considered that the formation of the canyons, by whatever mechanism, was greatly accelerated as a result of the Pleistocene glaciations. On the present hypothesis the sea exerts constraint upon the edge of the continental slab, and plastic equilibrium will be upset by an eustatic fall of sea-level. The effective increase in the height of the continents brought about by an eustatic fall of about 100 m, such as occurred during the glaciations, is comparable to several million years of normal increment. These relatively sudden disturbances would be accompanied by exceptionally rapid flow, for we have seen that in the process of flow by solution and recrystallization a fractional increase of stress may lead to a large increase of flow rate. Thus canyons could be produced even along margins that are ordinarily inactive.

The submarine canyons may thus have been formed by subaerial erosion and then carried out across the shelves. The steepness of the canyon sides may be a result of accelerated erosion during the glaciations, when sea-level was low. This would also help to explain the fact that not all canyons occur off present river mouths. The steepness of the sides would also tend to be enhanced by the process of downbuilding at the margin. Formerly one might have wondered that submerged canyons could remain largely clear of sediments, but this could undoubtedly be achieved by turbidity currents, whether or not they play an important part in eroding the canyon walls. Nor would transverse flow be fast enough to have squeezed the canyon walls together, as we can appreciate from the fact that guyots have subsided by only about 20 m since the beginning of the Pleistocene.

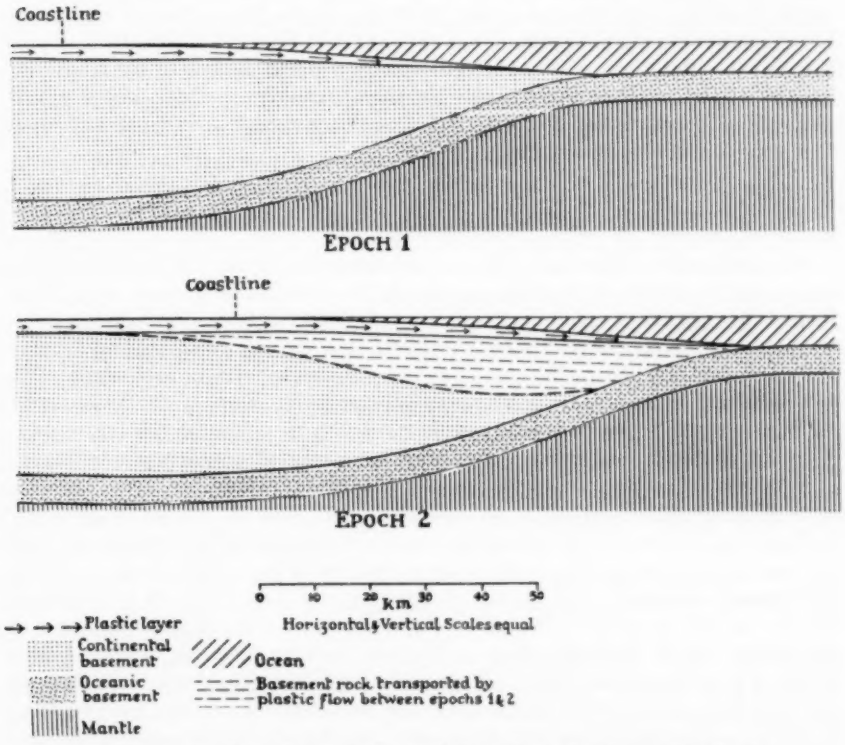

FIG. 10.—Continental growth by plastic flow of a surface layer. Idealized section across an active margin, omitting sediments. The depicted continental advance of 30 km between epoch 1 and epoch 2 requires a time of order 10–100 million years. The total length of plastic layer crossing the coastline in this time is about 400 km.

The process by which an active continental margin advances towards the ocean is illustrated in its essentials in Figure 10, and may be summarized in more detail as follows. A layer of basement rock to a depth of perhaps 2 km at the coast, and thinning out towards the bottom of the continental slope, flows steadily oceanwards carrying along its sedimentary burden, and to some extent maintaining its surface topography. This migration involves an increasing load on the substratum

and is therefore accompanied by steady subsidence; one is reminded of O. T. Jones's description of certain offshore regions as having a "sinking feeling" (Jeffreys 1952). Thus the adjacent oceanic crust is overwhelmed and warped downwards by the advancing edge of the plastic slab; the downwarping may produce an offshore trench, and in some cases faults may develop in the ocean floor parallel to the coastline. As more basement rock flows out on to the margin, the crust grows steadily thicker and a time will come when what was once typical oceanic crust has become fully continental. The pattern of strain that is imposed on the continental basement rock by the combined processes of plastic outflow and isostatic subsidence is beyond our present means to depict in detail, but there is ample scope here for producing the contortions that characterize gneissic shield rocks. Our picture of coastal regions as the site of important lateral as well as vertical movements seems to be at variance with the claim that former beaches, now recognizable at certain heights above sea-level, can be widely correlated, for this suggests that static coasts have been affected by eustatic changes of sea-level. The validity of the mode of formation of submarine canyons outlined above will be susceptible to critical test if reliable dates can be assigned to canyons and raised beaches situated along the same coast.

9. Marginal and oceanic disturbances

On the present hypothesis the rate of expansion of the continent is simply related to the velocity of the plastic layer. At the coastline we may say that a layer 2 km thick must achieve a crustal thickening of about 26 km; hence the normal rate of radial expansion of an average continent is about $v/13 = 0{\cdot}27 \times 10^{-6}$ km/yr. Let us assume that this value applies to the continental margin as a whole. The rate of subsidence may be taken as proportional to the slope of the Mohorovičić discontinuity, and this in turn as proportional to the slope of the sea bottom. Shepard (1948) gives 0° 07′ as the average inclination of continental shelves and 4° 17′ as that of continental slopes; the corresponding inclinations of the Mohorovičić discontinuity are about 0° 30′ and 16° respectively (cf. values of up to 10° in Figure 9). Hence the rate of subsidence is about 0·2 cm/1 000 yr under the shelf and 7 cm/1 000 yr under the slope. Along margins of more than average activity the rates of subsidence may be several times greater than these. It is of interest that for the 85 geosynclines listed by Kay (1955) the estimated average rates of subsidence are in the range 6–90 cm/1 000 yr, and the greatest estimated total thickness is 20 km. Thus the rate and amplitude of the disturbance that the process of continental growth launches into the substratum are comparable to those involved in geosynclinal sinking, or greater; and the area simultaneously affected by continental growth may be several hundreds of kilometres wide and many thousands of kilometres long.

This magnitude of disturbance may well have important effects both at the surface and at depth. One can only speculate upon the form that such effects might take. It is well known, however, that most of the major dynamic phenomena on the Earth's surface—including earthquakes, volcanoes and orogenies—are concentrated along certain continental margins. These phenomena are believed to arise from causes situated in the deep interior, but there has been no satisfactory explanation as to why they should coincide with margins of the continents. If now we have a further type of disturbance which can only occur at margins, namely the encroachment of continent upon ocean floor, the possibility emerges that this

disturbance and the seismic, volcanic, and orogenic activities are interrelated in a complex of cause and effect. We have already seen in other connexions that orogeny and volcanic outbursts are followed by a more or less rapid plastic flow. In advancing towards the ocean the flowing material upsets a primeval state of stability, and the resulting strains may well give rise to shallow earthquakes under the continental slope and, by accumulation over a long period of expansion, deeper ones inland. Conceivably these earthquakes may encourage further volcanic activity, while on van Bemmelen's (1954) theory the subsidence of the margin would eventually produce an orogenic reaction. Some fraction of the displaced mantle material will presumably in any case be driven landwards, there to cause uplift such as is indicated along many coasts by the occurrence of elevated marine terraces; this uplift, in combination with subsidence offshore, will produce the well-known effect of marginal flexure. It is not of course suggested that plastic flow at the margin could be sole cause of other marginal disturbances, but that it may at least serve to determine where they will occur.

The progressive displacement of some 20–30 km depth of mantle material by the advancing continental margin (Figure 10) can also hardly fail to produce conspicuous and far-reaching distortions of the ocean floor from below. We may picture an elastic oceanic crust and relatively viscous substratum being loaded at opposite sides by the adjacent continents. As the marginal regions are weighed down the pelagic region will tend to rise, and depending on the parameters of the system the resulting distortion might well resemble the oceanic elevations as we know them. The dominant feature of the Atlantic floor, for example, has been described as follows: "The Mid-Oceanic Ridge is a broad fractured arch whose axis follows the median line of the ocean. It generally covers the centre third of the ocean." (Heezen, Tharp & Ewing 1959.) The remarkable extent to which the crests of elevated zones in the oceans coincide with geometrical medians between the adjacent continents has been demonstrated by Menard (1958), who has already suggested the possibility of some causative relation between continental margins and oceanic rises and ridges. If this relation depends essentially upon continental expansion, as we now propose, it is perhaps not altogether surprising that the median elevations provide the second great zone of seismic and volcanic activity on the Earth's surface, only less disturbed than the marginal zone; for the elevations are the cumulative if somewhat remote effect of a process as old as the continents themselves.

We have already seen that no appreciable plastic layer is likely to be found under the oceanic abyssal plains, where the basement is overlain by $\frac{1}{2}$–1 km of sediments and about 5 km of water. But the crest of the Mid-Atlantic Ridge, for example, is raised to an average depth of about 3 km, so that conditions favouring solution and recrystallization must be expected to obtain in the basement to an average thickness of 1–2 km. In view of the long-standing stability of the oceanic crust, however, the onset of plastic flow will tend to produce the same kind of deformation as we have found in the case of continental shield regions, i.e. block faulting and the development of rift valleys. Several actual discoveries of rift valleys along the crest of mid-oceanic ridges are cited by Ewing & Heezen (1956) who have put forward the view that such median rift zones may extend continuously for great distances, and may even be primary features, with the median elevations as a by-product. The present hypothesis suggests, on the contrary, that rift valleys develop as a consequence of the formation of mid-oceanic ridges, and that well-developed rifts are unlikely to be found on the broader and less elevated

rises. Further, our interpretation implies that a seismic wave similar to the *Lg* phase should propagate along the Mid-Atlantic Ridge; unfortunately such a wave will not be easy to record.

The conception that median elevations originate simply as topographic irregularities in a normal oceanic crust seems to accord with the gravitational and seismic evidence. It appears that the normal condition of the crust is largely preserved under the median rises, but where the rise has developed into a ridge the basement layer may lose its identity. This important change is indicated on the Mid-Atlantic Ridge by the presence of a single seismic velocity of about 7·2 km/s, which lies between the values normally found on either side of the Mohorovičić discontinuity. Ewing & Heezen (1956) mention the possibility that the basement and mantle rocks have become physically mixed. The drastic upheaval that would be required in order to achieve this result seems a plausible culmination of the development outlined above, in which a thin crustal layer overlying a region of instability is seriously weakened by the opening of a rift. Lesser disturbances in the form of seismic and volcanic activity begin to appear at a much earlier stage; again we regard the location of this activity, but not necessarily its cause, as attributable to the process of continental growth.

10. Conclusion

The hypothesis that plasticity is a determining factor in the deformation of crystalline basement rocks at shallow levels seems to offer a coherent explanation of many major geological phenomena. It is almost wholly on this basis that the hypothesis must at present be assessed. The suggested process by which plastic flow takes place, that of solution and recrystallization, has been little studied either experimentally or theoretically, and further developments in this field would be of great interest. In discussing the geological evidence it has been possible to attain some degree of precision on several points by applying the theory of plastic equilibrium that Nye has developed in connexion with the flow of glaciers and ice sheets. On other points, however, a like precision must await a theory of plastic instability. Although deformations that are predictable by the equilibrium theory are exceedingly small, even these may on occasion be susceptible to direct observation. For example, two places situated on diametrically opposite coasts of a continent should recede from each other at an average rate of order 1 cm/yr, and in some circumstances at a rate of order 10 cm/yr; such movement cannot reliably be detected by standard geodetic methods, but may be identifiable over a period of a decade or so by the aid of the recently invented Danjon Astrolabe.

The present hypothesis depends on some means such as volcanism by which the average height of a land mass is increased from time to time. Thereafter the only force invoked is gravity, and the essential motions are located in the topmost few kilometres of the crust. Thus the hypothesis is more readily amenable to further test than are those in which internal motions are supposed to give rise to forces acting upon the crust from below. In the meantime, we are led to a strong affirmation of the view expressed by Wilson (1954) that "shelves, mountains, and shields . . . seem to represent successive stages in the growth of continents over former ocean floors".

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The Low-frequency Scaling Laws and Source Levels for Underground Explosions and Other Disturbances

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Summary

A general approach is used to show that the source spectrum level for the energy radiated from a disturbance is usually proportional to (frequency)^2 at low frequencies, and also that this low-frequency energy is proportional to (total energy)^2 for body waves. There are differences for one- and two-dimensional propagation, and also for interface waves. The application to underwater explosions is pointed out, using theory and experimental results recently reported by the author. The main application considered is that to underground explosions, and provides the best explanation of the empirical law that seismic amplitude is proportional to charge weight. An account is given of the reason why explosions underwater have much more low-frequency energy than those underground; typically efficiency is 10 per cent compared to 0·05 per cent, making underwater explosions more suitable as sources for seismic investigations. The general method is applicable to a wide range of other mechanical disturbances, and also to electromagnetic pulses.

1. Introduction

Surprisingly little has been published about the variation of radiated energy with charge size for underground explosions, and perhaps even less about their absolute source levels. It is commonly assumed that the radiated amplitude should be proportional to $W^{1/2}$ or $W^{1/3}$, where W is charge weight, although actually these laws hold only in severely limited conditions. It is shown here, using a very general approach with few assumptions, that the low-frequency energy spectrum level for many types of disturbance is proportional to (frequency)^2. It is further shown that, if the scaling is correct, this low-frequency energy level is proportional to (total energy)^2 for body waves, and to (total energy)$^{5/3}$ for interface waves. These results apply to a wide range of phenomena, and it is necessary to consider the detailed mechanisms involved only when discussing the limitations in the various applications.

The case of explosions fired underwater is taken first, since the mechanisms are well understood. Cole (1948) may be given as a general reference, and a recent paper by the author (Weston 1960) considers underwater explosions as acoustic sources. The theory in this last paper covers the whole frequency range, and although much of the experimental work is at relatively high frequencies some of it is relevant to the present discussion. Underground explosions, where the

mechanisms are less certain, are treated next—and their low-frequency levels compared with those from underwater explosions. Some of the other applications to mechanical and electromagnetic disturbances are included, but treated more briefly.

2. General theory for three-dimensional disturbances

Consider the time integrals $\int p\,dt$ and $\int u\,dt$, which are evaluated some distance outside the region of disturbance. Here p may be regarded primarily as representing the acoustic pressure and u the radial particle velocity, so that the integrals correspond to impulse and displacement respectively. However, the arguments below are quite general and still apply if p and u are oscillatory shear stress and tangential particle velocity, or even the variable parts of the electric and magnetic field strengths. If the integrals extend in time from the beginning of the pulse (or $-\infty$) to $+\infty$ it may be shown that they eventually fall off with distance faster than p or u themselves, so that in comparison at great distances they may be taken as zero. Some of the practical distance laws are shown in Section 4, Table 3. Care must be taken not to confuse the total impulse with another impulse value often quoted, for which the integration time extends from the beginning of the pulse to the first zero crossing.

Now there are some useful general relations between pulse shape and spectrum slope, illustrated in Table 1. The pulses are taken to be functions of time, and are derived from the pulse shape above them by differentiation. The table could have been written using the operator $D \equiv d/dt$, and it appears that if the lowest derivative that is zero at infinity is $D^m p$ (where m is allowed to be negative or zero as well as positive) the lowest derivative to show a discontinuity is $D^{(m-1)}p$ and the spectrum level is proportional to f^{-2m}. It is easiest to follow the table by starting in the middle at example 4, where the pulse shown is a Dirac delta function, well known as having Fourier amplitude coefficients independent of frequency f. The spectrum level (or "energy" in a 1 c/s bandwidth, proportional to $\int p^2 dt$), is also independent of frequency, for all frequencies. Each time there is a differentiation the relative spectrum level must be multiplied by f^2, to give the spectrum slope shown in the last column of Table 1, where the appropriate frequency range (high-frequency or low-frequency) is shown in the table in brackets. The pressure step in example 3 shows a spectrum level dependent on f^{-2}, which is also a well-known result, but the other examples are rather less common.

In the present case $\int p\,dt = 0$, which is true for examples 5, 6, 7 etc. Thus at low frequencies the spectrum level is proportional to f^2 or some higher power of f. It is not in general true that $\iint p\,dt = 0$ at ∞ so that normally the f^2 law may be expected. Nevertheless it is necessary to remember the possibility of a law with a higher index; which may for example occur at intermediate frequencies and give way to the f^2 law at very low frequencies (see practical example in Section 4, from Weston 1959); or may have a wider frequency validity if there are special propagation effects such as the presence nearby of a pressure-release boundary. In order to confirm f^2 as the expected law it is only necessary to know sufficient about the disturbance to rule out such special effects.

Thus for most disturbances at ranges greater than a few wavelengths the total impulse (or displacement) is virtually zero and the low-frequency source spectrum level is proportional to f^2. This is the first important general result, which applies to both body and surface waves.

For the next step the disturbance scaling laws are introduced, and it is necessary to restrict the discussion initially to body waves—though a free field need not be assumed. For example a disturbance in or very near a surface could be postulated, and it is unnecessary for the source to be symmetrical. In general throughout this paper W is used indiscriminately to mean either the total energy associated with a disturbance or the explosive charge weight. If there is proper scaling it is also implied for example that the type of explosive is unchanged, and the distinction does not matter. However the acoustic radiation from high explosives (various chemical or nuclear sources etc.) seems to depend surprisingly little on their nature, and almost entirely on the total energy release.

Table 1

Relation between pulse shape and spectrum slope

Example number	Schematic shape of typical pressure pulse p	Lowest derivative that is zero at ∞	Spectrum level proportional to
1		$\frac{d^3p}{dt^3}$	f^{-6} (H.F.)
2		$\frac{d^2p}{dt^2}$	f^{-4} (H.F.)
3		$\frac{dp}{dt}$	f^{-2} (H.F.)
4		p	indep. of f (all f)
5		$\int p dt$	f^2 (L.F.)
6		$\iint p dt$	f^4 (L.F.)
7		$\iiint p dt$	f^6 (L.F.)

Consider the changes when the disturbance is increased in size and everything scales properly. The time scale is changed as the linear dimension, i.e. as the cube root of the energy or say as $W^{1/3}$, and therefore f is proportional to $W^{-1/3}$. At the scaled range and scaled time the same values of p and u are obtained. The energy flux density in a scaled octave band (or any other constant percentage bandwidth) at the scaled range is proportional to $\int p^2 dt$ and therefore to t, and varies as $W^{1/3}$. Due to the $W^{-1/3}$ reduction in bandwidth the spectrum level at the scaled frequency and scaled range is proportional to $W^{2/3}$. Now the further assumption of a spherical spreading law is made, so that the energy correction from scaled to original range is as $W^{2/3}$. Thus spectrum level at the scaled frequency and original range is proportional to $W^{4/3}$, for all frequencies. Consider now only the low frequencies where the f^2 spectrum law holds, and the correction from scaled to original frequency is $W^{2/3}$. Hence spectrum level at the same frequency and same range varies as W^2.

This may be stated alternatively as low-frequency energy proportional to (total energy)^2, or as low-frequency amplitude proportional to total energy. It is the second important general result, and applies to elastic longitudinal or shear waves and to electromagnetic body waves. In passing it may be pointed out that the

low-frequency energy to which the f^2 and W^2 laws apply forms only a small part of the total vibrational energy of the disturbance.

It is necessary to investigate the mechanisms of the various disturbances and to check the validity of the assumptions only when it is desired to establish the frequency range in which the laws apply. Thus in general the lower frequency limit is set by the necessity for the measurement point to be at several wavelengths' range, so that the impulse and displacement may be effectively zero and so that the spherical spreading law may apply. The upper frequency limit is set by the requirement that the period should be much greater than the total pulse duration, so that the relations between pulse shape and spectrum slope may hold. It may be noted in passing that due to the usual source spectrum shape the effect of approaching the upper frequency limit will commonly be a tendency to reduce the index in the W^2 law; but there is the possibility of having a low-frequency slope nearer to f^4 than f^2 and therefore a tendency for the index in the W^2 law to increase. It may also be shown that the effect of appreciable attenuation with range at and above the upper limit of the f^2 law will be to reduce this limit, while the limit of the W^2 law is unaffected. Special effects due to large amplitudes should usually be unimportant in this application.

3. Theory for one and two dimensions and for interface waves

It is interesting to consider briefly what happens in an N-dimensional medium, when the basic linear scaling factor becomes $W^{1/N}$. With the one exception noted below the impulse or displacement may still be taken as zero, and the f^2 spectrum law will apply. The geometrical spreading law is energy proportional to $(\text{range})^{-(N-1)}$. It is found using the arguments of Section 2 that the low-frequency spectrum level at the same frequency and range varies as $W^{(N+3)/N}$. In possible real cases $N = 3$, 2 or 1 giving variation with W^2, $W^{5/2}$ and W^4 respectively.

The exception can occur in one-dimensional propagation, since p, u, $\int p dt$ and $\int u dt$ do not fall off with range. However the two integrals may still be zero everywhere—e.g. the struck end of a semi-infinite rod may return to its original position so that displacement $\int u dt = 0$ and therefore $\int p dt = 0$. In this case the f^2 and W^4 spectrum laws will both hold at low frequencies. In general there will be a finite displacement, corresponding to example 4 of Table 1, and giving at low frequencies a spectrum level independent of frequency and proportional to W^2.

An important practical case concerns the interface wave generated by a three-dimensional disturbance, such as an underground explosion in or near the surface. It should be stressed that the two-dimensional calculation above does not apply here. It is necessary that everything scale correctly, e.g. the explosion depth must vary as $W^{1/3}$, when the appropriate argument closely follows that in Section 2. The only difference is that such waves follow a cylindrical spreading law, so that the level correction from scaled to original range is $W^{1/3}$. Thus in scaled conditions the low-frequency level for interface waves varies as $W^{5/3}$.

There are a few other cases, though not of any great practical importance, and these are included with the other results in the summary in Table 2. It may be noted that at long ranges the displacement or impulse must be zero for the interface wave in the two-dimensional geometry, and this case is analytically the same as that for the "tube wave" in two dimensions when there is no displacement.

4. Underwater explosions

As mentioned in the Introduction, underwater explosions are now well understood (Cole 1948, Weston 1960). A measure of the total source duration is given by the time to the first bubble pulse, and the low-frequency laws deduced above will therefore apply for frequencies well below the reciprocal of this time:

$$F_b = 0{\cdot}23 d_0^{5/6} W^{-1/3} \text{ c/s}. \qquad (1)$$

Here d_0 is the depth in feet plus 33, corresponding to the total pressure, and W is the weight of TNT in lb.* A maximum of the spectrum level curve occurs at the bubble-pulse frequency F_b and a typical value is 15 c/s for a 1 lb charge at 20 fathoms depth (0·45 kg at 37 m).

Table 2

Summary of frequency and energy-dependence of low-frequency spectrum level

	Frequency dependence	Energy or weight dependence 1-dim.	2-dim.	3-dim.
Tube wave, finite displace.	indep.	W^2	W	$W^{2/3}$
Tube wave, zero displace.	f^2	W^4	W^2	$W^{4/3}$
Interface wave	f^2	—	W^2	$W^{5/3}$
Body wave	f^2	—	$W^{5/2}$	W^2

Weston (1960) gives a theoretical expression which the spectrum level approaches asymptotically at low frequencies, and which is proportional to f^2 as expected. The coefficient of f^2 is surprisingly low, due to the shape of pressure pulse, which is more like example 6 than example 5 of Table 1. The full theoretical curve shows a rapid fall-off just below F_b, where the local law is in fact close to f^4. Because of these facts it appears theoretically to be necessary to go about two decades below F_b before the f^2 spectrum law will apply accurately. In addition a rather variable level may be expected in practice, since the rapid fall-off is due to a cancellation of large terms which need change only a little to produce appreciable effects in the low-frequency levels, due to a redistribution of the energy in the frequency spectrum (see Weston 1960).

It is convenient at this point to give in Table 3 the range dependence laws for p, u, $\int p\,dt$ and $\int u\,dt$ for underwater disturbances in order to illustrate the relatively rapid fall-off for the integrals. The comparable laws for underground explosions are included, based on the simplest of the mechanisms described in Section 5.

There are very few experimental results apart from those of Weston (1960), and many of those are at high frequencies. Evidence is presented there for the existence of the spectral maximum at F_b (octave measurements of the absolute source levels of detonators, octave measurements of the differences between detonators and 1 lb (0·45 kg) charges, measurements of the level variation with depth for a 1 lb charge at 10·8 c/s). However, none of the source spectrum measurements extend quite to the region of the expected f^2 law, and neither do the measurements of differences between charges extend into the (same) region where

* Equation (1) coefficient is 0·475, for d_0 depth in metres plus 10, W in kg.

the W^2 law should hold. The measurement nearest to this region is that of 45 dB between detonators and 1 lb charges for the 35 c/s octave at 30 fathoms (55 m) depth compared to the W^2 law value of 54 dB. As expected theoretically the measured difference was increasing with decreasing frequency and increasing depth.

Table 3

Geometrical spreading laws for underwater and underground disturbances

The changeover from the close to the distant spreading law occurs for range r of the order of one wavelength. The integrals are both zero if there is no overall introduction of material, and also $\int p\,dt = 0$ always for fluids. The small-amplitude approximation is made.

	p close and $\int p\,dt$	u close and $\int u\,dt$	p distant and u distant
Fluid	r^{-1}	r^{-2}	r^{-1}
Solid	r^{-3}	r^{-2}	r^{-1}

5. Underground explosions

Since this subject is rather less well understood than that of underwater explosions it is convenient to start with some of the experimental data. The writer knows of no quantitative evidence either for or against the validity or the f^2 low-frequency spectrum law. A few workers have reported investigations of the scaling laws, however, and the law that seismic amplitude A varies as charge weight W is a reasonable fit to all results. O'Brien (1957 and 1960) has summarized most of the experimental observations made by Habberjam & Whetton (1952), Gaskell (1956) and himself. Excluding close-range results (which may contain appreciable high-frequency energy) he found the best fit in the relation $A \propto W^n$ was given by $n = 0{\cdot}98$. On re-analysis the results of Shock (1950) may be seen to be fitted quite well by a similar relation, which was also obtained recently by Latter & others (1959) for nuclear explosions.

The $A \propto W$ law above is of course the same as the W^2 law for low-frequency energy derived in Section 2. It is necessary to add that in seismic prospecting at any typical range the high frequencies are attenuated, and only the low-frequency energy is measured. In addition the signal duration and shape are governed by the propagation conditions, assumed constant, so that the instantaneous amplitude is proportional to the square root of the total energy in the band. By its nature the W^2 law may explain quarry-blasting results with shallow shots (Habberjam & Whetton 1952, etc.) as well as deep shots, provided it is body waves (e.g. reflected waves) that are received and provided the shot depths etc. all scale. However the seismic efficiency of shallow shots seems to be much worse than that of deep shots of the same size. If interface wave energy is measured the theoretical law is changed only slightly to variation with $W^{5/3}$ for energy, i.e. $A \propto W^{5/6}$ (Section 3), and experimentally this would be hard to distinguish from the W^2 law. However, there is scaling trouble with all shallow shots, since the physical properties of the medium usually vary rapidly with depth, and the practical laws may therefore differ from the theoretical laws above. It may be noted that a charge lacking central symmetry will generate a certain small amount of shear wave energy, to which the W^2 law will still apply.

It is necessary now to consider what happens when a charge explodes underground. The supposed process for deep charges has been described by Sharpe (1942), Morris (1950) and others. As the explosive pressure pulse spreads out into the surrounding material it produces a compression plus a lateral strain, and at close ranges the latter is large enough to fracture the rock, energy being lost in the formation of a complicated crack system. This shattering of rock extends out to a critical radius R beyond which there is the normal elastic wave propagation. Calculations may be made applying spherical wave theory in a solid medium, outwards from the surface of an "equivalent radiator" having this critical radius. Thus it may be shown, e.g. from Blake (1952) that the relation between r.m.s. pressure at a distance and that, p_R, at the critical radius is given by

$$p = k^2R^3p_R/r[1-k^2R^2+k^4R^4]^{1/2} \qquad (2)$$

where k is the phase constant and for simplification a Poisson's ratio of $\frac{1}{3}$ has been assumed. This model is supported by a number of empirical observations which are listed by Sharpe (1942). Modifications to this description are necessary for some solids, for example in clay there is plastic yielding rather than shattering. In practice the conditions near the source are likely to be very complicated, and in general there may for the same shot be regions of rupture and of plastic flow.

O'Brien (1957 and 1959—see also Latter & others 1959) has given a theoretical explanation for the $A \propto W$ law based on the above model, pointing out correctly that seismic amplitude at a distance is proportional to R^3 (equation (2)) and thus to W provided the pressure on the face of the equivalent radiator is constant. However he does not discuss the meaning of this pressure constancy, and of course the whole explanation depends on the correctness of the model and thus applies only in limited conditions. Ricker (1951) and Gaskell (1956) have suggested different reasons for a theoretical law $A \propto W^{5/6}$, though on examination there does not seem to be any real justification for either argument. The fallacy in Ricker's theory arises from the subtle point that it is not allowable to apply the normal linear scaling laws (with $W^{1/3}$) when there is appreciable dissipation with attentuation coefficient proportional to f^2 (as opposed to f).

The approach preferred here is to start with the generally applicable f^2 and W^2 laws, assume the model, and work back to the source conditions. The simple theory expressed in equation (2) shows that at low frequencies the ratio of distant pressure spectrum level to the close level at range R is proportional to f^4 (*N.B.* $f \propto k$ and spectrum level $\propto p^2$), so that the f^2 law at a distance corresponds to the f^{-2} law at the critical radius R. It is in fact only with this f^{-2} law that the spectrum level at a constant frequency and scaled range R is independent of charge size. The constancy of pressure mentioned above means a pressure spectrum level independent of charge weight but not independent of frequency. The simplest pressure–time curve which will give the f^{-2} law is the pressure step, illustrated in Table 1 example 3, and it may be seen that the assumed model practically demands this form since there is no way postulated for the explosion gases to escape. In practice there will be high-frequency oscillations superimposed on the beginning of the step, and in addition any slow pressure decay due to the escape of the explosion gases will not matter provided the decay period is long compared to the period of the slowest seismic waves. If the decay has a period comparable with or faster than the seismic wave period the f^2 and W^2 laws will still in general hold, but the spherical elastic wave theory may not apply right in to the critical radius and it may be necessary to abandon the model. As discussed in Section 2 it is

theoretically possible to have laws faster than f^2 and W^2 (say f^4 and therefore $W^{8/3}$) for very special kinds of disturbance. This could be reconciled with a pressure decay in the above model since it would lead to a spectrum level at the critical radius independent of f and proportional to $W^{2/3}$.

The low-frequency limit for the f^2 and W^2 laws will depend in practice on the measurement geometry, but even in an infinite solid medium it is necessary that the measurement range should be much greater than one wavelength, and it would also be interesting to know about the pressure decay rate. However this limit is of little concern, whereas the upper frequency limit is of great importance. Now if a pressure step is applied to the spherical face of the equivalent radiator a very highly damped sinusoidal transient is propagated, having a phase constant $k \sim 1/R$ (Blake 1952). The damping constant is numerically equal to about half the angular frequency, and the transient lasts about as long as the shock-wave pressure pulse from an underwater explosion of the same size (though the precise relation depends of course on the underground medium). For a typical crystalline rock one may take, say, longitudinal velocity $c = 12\,000$ ft/s (3·66 km/s) and $R \sim 2W^{1/3}$ ft where W is in lb.* Thus the frequency of the transient $F_s \sim 1\,000W^{-1/3}$ c/s*, or about 1 kc/s for a 1 lb (0·45 kg) charge. The radiated spectrum at close ranges should show a maximum at this frequency, and for frequencies much less than this the f^2 and W^2 laws should apply. It is now possible to calculate the minimum range at which the $A \propto W$ law should apply, since A is the seismic amplitude (of displacement or particle velocity) measured broadband. The attenuation in the rock is likely to be about 0·1 dB per wavelength; i.e. 6 dB down after 60 wavelengths. At the frequency of the transient this corresponds to range $r \sim 700W^{1/3}$ ft (or weight $W \sim 3 \times 10^{-9}r^3$)*; and barring special propation effects it is only for distances much greater than this that all the energy remaining is in the low-frequency part of the spectrum, and the $A \propto W$ law holds accurately. Thus for large charges or close ranges a smaller dependence of A on W would be expected. It must be remembered that these figures may be changed appreciably for other types of material, such as clays, controlled by different factors.

6. A comparison of the low-frequency energy from explosions underwater and underground

Before considering details it is worth while pointing out that there are many similarities between explosions in different media. For example the quantity of radiated energy is about the same for underground, underwater and aerial explosions—though the decay rates are different. Perhaps the most important difference between underground and underwater explosions arises because for the latter the restoring force is due to the hydrostatic pressure (instead of the rigidity modulus), leading to a negative pressure phase of much greater duration than the shock wave preceding it. Thus below F_s the underwater explosion has a flat spectral region, which is not present in the underground case. Figure 1 compares the main spectral features at close ranges, though with considerable simplification.

With the model assumed (the equivalent radiator with a simple pressure step at R) it is possible to calculate the whole spectrum for underground explosions, though this cannot be done very precisely because of considerable uncertainties in

* Or $R \sim 0{\cdot}8\ W^{1/3}$ m, $F_s \sim 800W^{-1/3}$ c/s, $r \sim 300W^{1/3}$ m for W in kg, and $W \sim 5 \times 10^{-8}r^3$ for r in m.

the values of the parameters. It is easiest to start with the f^{-2} law for the pressure spectrum at R and convert to a distant range using equation (2). The spectrum level of energy flux density for a plane-wave pressure step (compare methods of Weston 1960) is

$$E(f) = P_0^2/2\pi^2\rho c f^2 \tag{3}$$

so that in the present case

$$E(f) = \frac{2P_0^2R^4}{\rho c^3 r^2}[k^{-2}R^2 - 1 + k^2R^2]^{-1}. \tag{4}$$

If one guesses the pressure step at the critical radius $P_0 = 300$ atmospheres (3×10^8 dyne/cm^2), and takes $R = 2$ ft (0·6 m) for 1 lb (0·45 kg) charges, density $\rho = 2{\cdot}6$ gm/cc, $c = 12\,000$ ft/sec (3·66 km/s) and $r = 100$ yd (91 m) (to agree with curves for underwater explosions given by Weston 1960) the symmetrical spectrum of Figure 2 results. It may be noted that P_0 is not the same as the shock wave

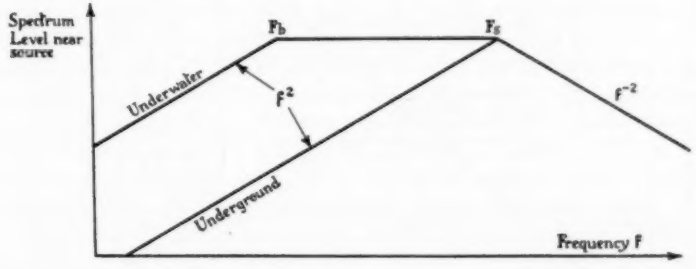

FIG. 1.—Relative spectra for explosions in different media, much simplified.

peak pressure at 2 feet range, and since the model does not take account of this the levels at and above $F_s = 955$ c/s are likely to be lower than the measured levels. They would agree perfectly with the high-frequency underwater levels if a figure of say 1 000 atmospheres for the shock pressure were used in equation (4), instead of 300 atmospheres for the pressure step P_0. The shock pressure at the critical radius is determined by the strength of the rock, and in turn determines the value of P_0. This discrepancy may be swamped by high-frequency attenuation effects, the typical magnitudes of which are also shown, and are seen to be large even at 100 yd. The spectrum is likely to be nearer the truth for the lower frequencies. However the Figure 2 spectrum should perhaps be regarded as an illustration of a particular theoretical model rather than as a serious attempt to predict levels accurately.

If the very-low-frequency region below F_b is neglected, examination of Figure 1 suggests that at a given low frequency the spectrum level for explosions underwater exceeds that for those underground by the factor $(F_s/f)^2$—or about 25 dB at 50 c/s for a 1 lb charge. Comparison of the detailed spectra of Figure 2 and of Weston (1960) shows a larger difference for various reasons (mainly enhancement of underwater levels due to the bubble pulse, also F_s is not quite the same underwater and underground), i.e. about 40 dB at 50 c/s, and the real value probably lies in between. If frequencies up to only f are present Figure 1 suggests the total energy in the underwater case is proportional to f and in the underground case to f^3, with ratio $3(F_s/f)^2$. Gaskell (1956) has quoted his experimental impressions

that explosions in clay are 3 to 4 times more effective than those in the worst rocks, and again that explosions in water are about 4 times better than those in clay. The overall amplitude ratio for water to rock is thus about 14, or 23 dB. These figures are for no specified charge size or frequency, but for a typical 50 lb (23 kg) charge with frequencies up to 50 c/s the (optimistic) Figure 1 approach leads to a difference of 18 dB, with probable real value therefore about 25 dB. This agreement together with the general account given here seems to establish beyond doubt that the better seismic results obtained underwater are primarily due to the much greater efficiency of the underwater explosions as low-frequency sources.

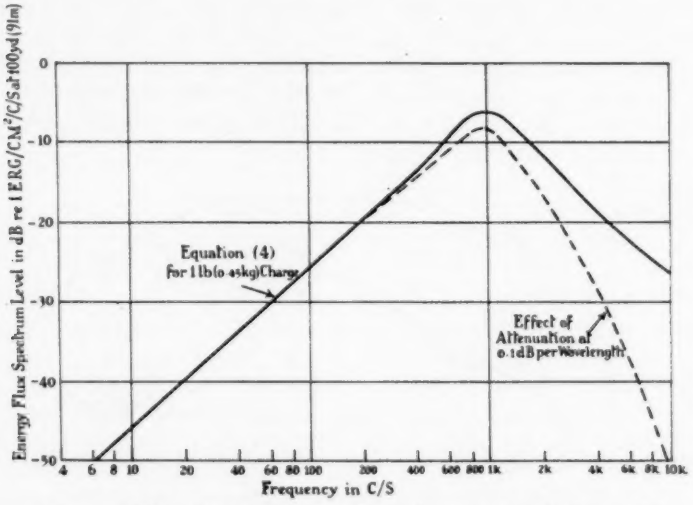

FIG. 2.—Theoretical source level for underground explosion model.

The calculations and measurements reported by Weston (1960) enable the reliable calculation of the absolute efficiency of underwater explosions as low-frequency sources; e.g. for a 50 lb charge up to 50 c/s the radiated energy is about 10 per cent of the chemical energy, though for the whole frequency band it approaches 50 per cent. This figure depends a little on the charge depth, and would be reduced by more than half if there were no bubble pulse (which is effectively the case at shallow depths due to migration). The comparable figure underground is probably of the order of only 0·05 per cent! O'Brien (1960) has summarized the various experimental estimates of the pulse energy underground, unfortunately they are all for close ranges and therefore include the higher frequencies. In view of this there is no discrepancy between the typical reported figure of 1 per cent and the 0·05 per cent above. A plea is entered here for observers to report measurements in reasonably narrow frequency bands (an octave band is a good compromise), since the scaling laws and absolute energies depend critically on frequency, and wide-band results are hard to interpret.

It may be noted that the efficiency of charges underground increases rapidly with size till F_s has fallen to about 100 c/s, and this occurs at about $W = 1\,000$ lb (450 kg). This weight actually depends on the upper limit of the frequencies propagated, and the formula in Section 5 gives $W \sim 3 \times 10^{-9} r^3$ in order of magnitude. In other words for typical ranges the $A \propto W$ law holds only for charges

appreciably smaller than this, as already discussed. Thus for maximum efficiency it pays to put all the explosive into one charge till the limiting weight is reached, after which the explosive should be split up into a number of charges each approximately equal to the limiting weight. It should be remembered that it is not the rate of increase of A beyond 1 000 lb that is poor, so much as the rate below 1 000 lb that is exceptionally good. In attempting to detect underground explosions at very long ranges the frequencies transmitted may be so low that even very large charges have a low seismic efficiency, which is of current interest in connexion with the monitoring of nuclear explosions underground. There are similar relations to these for underwater explosions, but the effects are not so marked because the spectrum shape is less peaked and in addition the propagated frequencies may be higher.

7. Other disturbances

The application of the f^2 and W^2 laws to underground, underwater and aerial explosions has just been considered, and some of the other applications will now be discussed briefly. The f^2 law applies very widely; but for various reasons including the difficulty of achieving proper scaling the W^2 law does not always follow, though there may well be dependence on some relatively high power of W.

Single-discharge spark sound sources in both liquids and gases behave almost exactly as do small explosive charges of equivalent energy. Thus a one-joule discharge corresponds to 5×10^{-7} lb ($2{\cdot}3 \times 10^{-4}$ g) of explosive (much smaller than a match-head), with F_g about 100 kc/s and for an underwater spark F_b about 1 kc/s. The f^2 and W^2 laws may both be applied here, and also to the comparable problem of noisy cavitation bubbles.

With thunder the upper frequency limit of the f^2 law is determined by the length of the discharge path, i.e. the size of the acoustic source, and also by the number of strokes, and may even lie below 1 c/s. The scaling difficulty stops the W^2 law applying, which would anyway be only at very low frequencies.

The typical sound source in air or water has a more complicated wave-form than those already considered, e.g. biological noises such as a fish drumming or a man speaking. With waveforms like that of Table 1 example 7, or a longer pulse of sinusoidal vibration, the f^2 law is likely to hold at such a low frequency that its occurrence is of academic interest only.

Earthquakes generate longitudinal and shear waves, and for seismic periods longer than perhaps several seconds the f^2 law may well hold. This limiting period is estimated with tectonic earthquakes in mind, and might be shorter for volcanic earthquakes. It may also hold for tsunamis, but perhaps only for periods longer than several minutes. As for thunder the W^2 law cannot be used, but it is worthwhile noting the possibility of overestimating the ratio of the total energies released in two earthquakes if all measurements are made on long-period waves or with a long-period seismograph.

The f^2 and W^2 laws could also apply to various artificial sources, such as the dropping of weights onto the ground. The f^2 law would apply to the surface waves on water due to the dropping in of some object, but not the W^2 law since the constancy of gravity prevents proper scaling.

The f^2 and W^2 laws may be applied to the electromagnetic radiation from sparks, the frequency limitation depending mainly on the discharge circuit. The f^2 but not W^2 law could apply to lightning discharge, with an upper frequency limit

of perhaps 1 000 c/s if there is a single mainstroke, going down to a fraction of 1 c/s for the longer multiple flashes.

8. Conclusions

At low frequencies the f^2 spectrum law is widely applicable, and the W^2 law applies commonly to body waves. However other f and W laws may also occur.

For underground explosions the f^2 and W^2 laws apply for frequencies much less than $1\,000W^{-1/3}$ c/s* for W in lb. This proves the $A \propto W$ relation, but shows that it only holds for range greater than about $700W^{1/3}$ feet.*

On comparing the spectra of underground and underwater explosions it is found that as a seismic source a typical explosion underground is about 25 dB weaker than one of equal size underwater, and has an absolute efficiency of only about 0·05 per cent. To obtain a reasonable efficiency charges of about 1 000 lb (450 kg) are needed.

Admiralty Research Laboratory,
Teddington,
Middlesex.
1959 *July.*

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* Or $800W^{-1/3}$ c/s and $300W^{1/3}$ m for W in kg.

Rock Magnetism and the Origin of the Midland Basalts

C. W. F. Everitt

(Received 1959 November 13)

Summary

There has been considerable dispute about the origin of the series of Carboniferous igneous formations in the neighbourhood of the Clee Hills, Shropshire. Palaeomagnetic measurements suggest that all the formations except one are intrusive. They also indicate that the accepted geological interpretation of one formation may require revision.

1. Introduction

There has been considerable dispute amongst geologists about the origin of the series of Carboniferous igneous formations, usually called the Midland basalts, lying in the area around the Clee Hills, Shropshire (Figure 1). For a long time they were regarded as sills or dykes, but in 1931 Pocock assembled considerable evidence suggesting that they were lavas. Later Marshall (1942) argued strongly for an intrusive origin for all the formations except Little Wenlock. The most recent discussion is by Whitehead and Pocock (1947), who endorse some of Marshall's conclusions but regard the formations at Kinlet and Shatterford as probably extrusive.

For most of the formations there is very little positive evidence to support either view of the origin. The overlying sediments show little sign either of induration or of derivation from the basalts; and the basalts themselves are not obviously either chilled or weathered.

The work to be described here suggests that measurements of natural remanent magnetization may be useful in such cases. The remanent magnetization in sedimentary rocks is usually very weak with wide scatter of direction about the mean; but if the material undergoes heating to a few hundred degrees centigrade it becomes remagnetized with perhaps 50–100 times the intensity and small scatter. This increase of intensity arises primarily from the difference in process of magnetization; that is to say, the ratio of natural intensity to the saturation intensity is higher in a baked rock than in an unbaked one.

It follows that ancient sills can often be distinguished from lava flows by examining the remanent magnetism of the overlying sediments, even when there is no apparent metamorphism. Above a sill the contact sediments should be magnetized in the same direction as the igneous body, and there should be a systematic increase of scatter and decrease of intensity with distance from the contact. Above a lava the sediments should usually have a different magnetic direction from the igneous body and have uniform intensity and scatter.

Complications may arise if there are variations in the proportion of magnetic material with distance from the contact; but they should be resolved by measuring

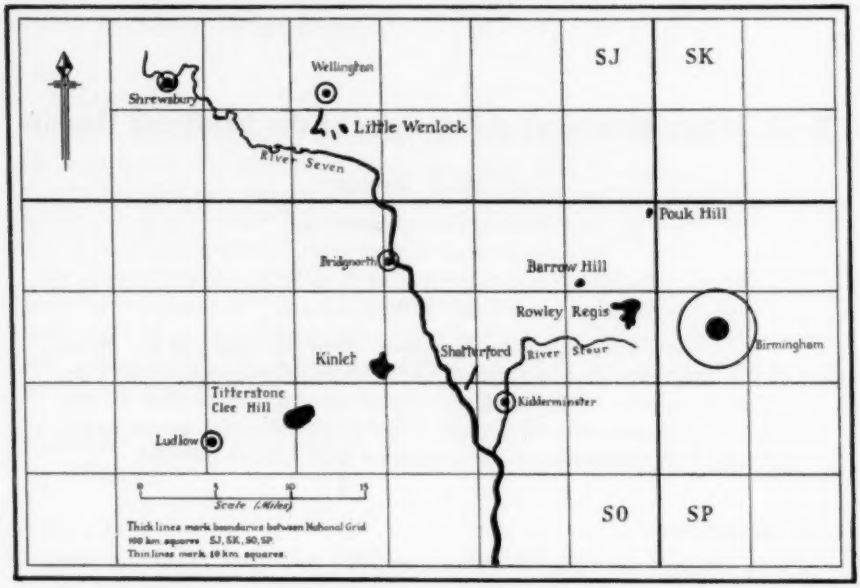

FIG. 1.—Distribution of sites.

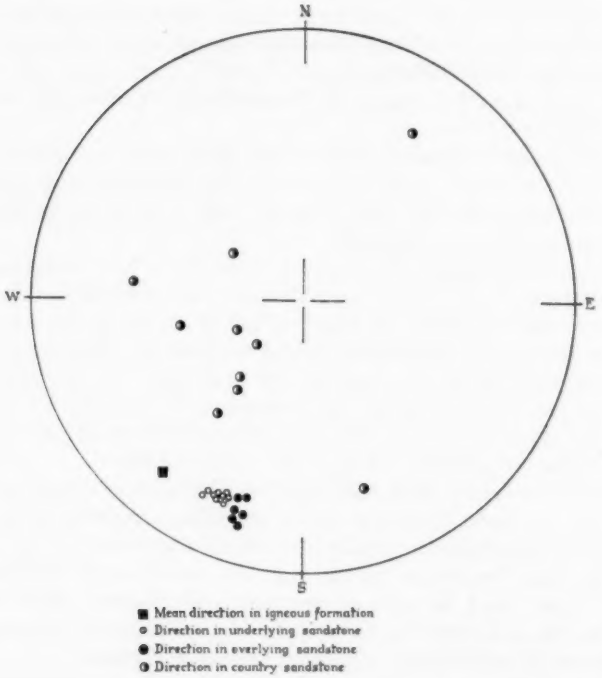

FIG. 2.—Results from Titterstone Clee Hill.

the ratio of the natural intensity to the intensity after saturation in a high magnetic field.

2. Origin of Midland Basalts

The overlying contacts have been examined at all the outcrops in the Midlands except Barrow Hill and Rowley Regis, where the material was unsatisfactory. Measurements were made on a Blackett magnetometer, using disks $1\frac{1}{8}$ in. diameter $\frac{1}{4}$ in. thick cut from oriented samples of rock. Table 1 gives the sampling details; the magnetic declinations and inclinations relative to bedding planes; the intensities of magnetization; and numerical estimates of the mean scatter of direction in each block, calculated from a formula due to Wilson (1959).

The directions for the site at Titterstone Clee Hill, which may be taken as typical, are plotted on the Schmidt projection in Figure 2. In the country sandstone 200 ft from the contact, the intensity was $0{\cdot}16 \times 10^{-6}$ c.g.s. units/g and the directions were widely scattered. In the sandstone within a few inches of the top and bottom contacts, the intensities were respectively 30×10^{-6} and 10×10^{-6} c.g.s. units/g; while the directions were closely grouped and agreed well with the mean direction in the igneous body. At an intermediate point five feet above the top contact the intensity was 3×10^{-6} c.g.s. units/g and the scatter lay between those of the contact and country rocks. Thus the intensity, scatter and direction of magnetization in the overlying sediments all suggest that the basalt is intrusive.

At Kinlet and Shatterford the overlying sediments again had high intensity and small scatter and agreed in direction with the igneous body. At Pouk Hill the intensity, as well as the scatter, was higher in the sample further from the contact; but this was simply due to differences in the magnetic material, for the ratio of natural to saturation intensity decreased from 0·7 per cent at 7 ft from the contact to 0·1 per cent at 15 ft. Therefore the measurements indicate an intrusive origin for each of these sites.

At Little Wenlock on the other hand the overlying sandstones were weakly magnetized with wide scatter, so the igneous body is probably a lava. The mean direction at Little Wenlock was 353° east of true north and 7° down as against 202° east and 17° down for the other sites. This also points to a difference of origin.

3. Geological aspects of the intrusions

The beds at Clee Hill and Kinlet are slightly folded and those at Shatterford extensively so. It is possible to learn something about the original geological aspects from the palaeomagnetic measurements.

The Clee Hill formation is shaped like a saucer. The magnetic directions from opposite points on the rim agree closely relative to the bedding planes but diverge by about 10° relative to the present horizontal. Since the directions seem to be very reliable, I am inclined to regard the difference as significant and conclude that the saucer was formed after intrusion. At Kinlet also, the directions agree more closely relative to the bedding planes, so again folding probably occurred after intrusion.

At Shatterford the problem is more complicated. At first sight the igneous body appears to be a dyke, a few feet wide and about two miles long. However since it lies parallel to the surrounding beds, which are steeply tilted, it may really

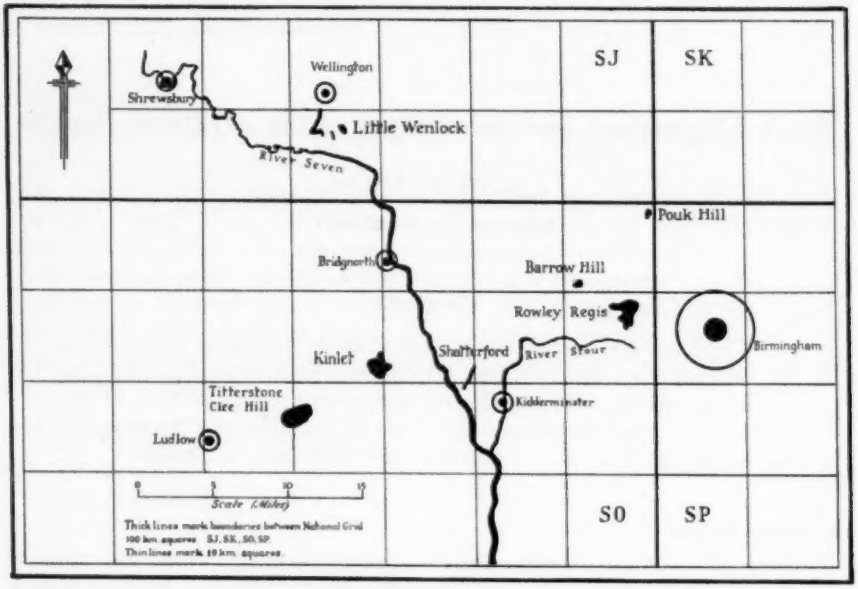

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Table 1

Site	Type of rock	Distance from contact	Magnetic declination (deg. East of true North)	Magnetic inclination	Dispersion	Natural intensity $\times 10^{-6}$ c.g.s. units/g	Sampling details: Site	Nat. Grid. Ref.	No. of blocks	No. of disks
Titterstone Clee Hill	Igneous	—	222°	26° down	24°	700	Clee Hill Southern Quarry	SO 595 764	2	5
	Underlying sandstone	2 ft	203°	22° down	1°·5	10	Titterstone Clee Hill Quarry	SO 594 778	1	12
	Overlying sandstone	6 in.	197°	20° down	3°	30	Magpie Quarry	SO 615 774	1	6
	Overlying sandstone	5 ft	197°	25° down	10°	3	Magpie Quarry	SO 615 774	1	6
	Country sandstone	200 ft	240°	64° down	26°	0·16	Benson's Brook	SO 584 772	2	10
Kinlet	Igneous	—	182°	18° down	—	1 400	Mass House Quarry	SO 703 821	1	6
	Underlying sandstone	2 ft	202°	17° down	4°	27	Raggits Quarry	SO 694 816	1	6
	Overlying sandstone	6 in.	210°	7° down	2°	36	Mass House Quarry	SO 703 821	1	6
	Country sandstone	30 ft ?	187°	0°	26°	0·8	North-west Quarry	SO 697 825	1	12
Shatterford	Igneous	—	30°	5° down	—	1 000	Witnells End	SO 786 817	2	14
	Igneous	—	215°[1]	1° down[1]	9°	15	Stream section	SO 785 815	1	12
	Underlying shale	6 in.	196°[1]	15° down[1]	5°	5·5	Stream section	SO 785 815	1	6
	Overlying shale	6 in.	204°[1]	17° down[1]	2°·5	14	Stream section	SO 785 815	1	7[2]
	Country material	not sampled	—	—	—	—	—	—	—	—

Barrow Hill	Igneous	—	208°	12° down	—	—	Quarry Ic[3]	SO 915 817	1	2
	Underlying material	not sampled	—	—	—	—	—	—	—	—
	Overlying material									
	Country material									
Pouk Hill	Igneous	not sampled	—	—	—	—	—	—	—	—
	Underlying material									
	Overlying black shale	7 ft	194°	13° down	3°	0·5	Bentley's Quarry	SO 994 994	1	6
	Overlying mudstone	15 ft	205°	1° down	24°	1–100	Bentley's Quarry	SO 994 994	1	12
Rowley Regis	Igneous	—	168°	15° down	20°	1 400	Element's Quarry	SO 969 884	2	10
	Underlying	not sampled	—	—	—	—	—	—	—	—
	Overlying									
	Country									
	Sandstone at side of body	1 ft	189°	0°	6°	17	Hailstone's Quarry	SO 968 884	1	6
Little Wenlock	Igneous	—	356°	15° down	—	270	Doseley Quarry	SJ 674 068	2	20
	Underlying sandstone	6 in.	10°	13° up	2°	1·2	Doseley Quarry	SJ 674 068	1	2
	Underlying limestone	2 ft	349°	1° up	14°	0·4	Lydebrook Dingle	SJ 658 063	1	12
	Overlying sandstone	1 in.	200°	70° down	45°	0·7	Lydebrook Farm	SJ 658 069	2	8
	Country sandstone	?	347°	22° down	25°	0·2	The Hatch	SJ 645 083	2	5

(1) Assuming beds not inverted, see Section 3 and Figure 4.
(2) This material, being very fissile, had to be measured in lumps about the size of a fist.
(3) Marshall's notation, see *Quart. J. Geol. Soc. Lond.*, **101**, 177, (1946).

be a sill that has undergone extensive folding after formation. Most of the samples were collected either at Witnell's End or at the stream-section 300 yards south of Witnell's End (see Figure 3). Between these two sites the tilt direction changes by about 90°, implying that the sill has undergone helical folding into the form of a propeller-blade. The fold cannot be traced in detail, since the intervening material, although marked by a prominent ridge of land, is not actually exposed.

The magnetic directions at the two exposures are plotted relative to the present horizontal in Figure 4(a). They are almost opposite. Since the exposures appear

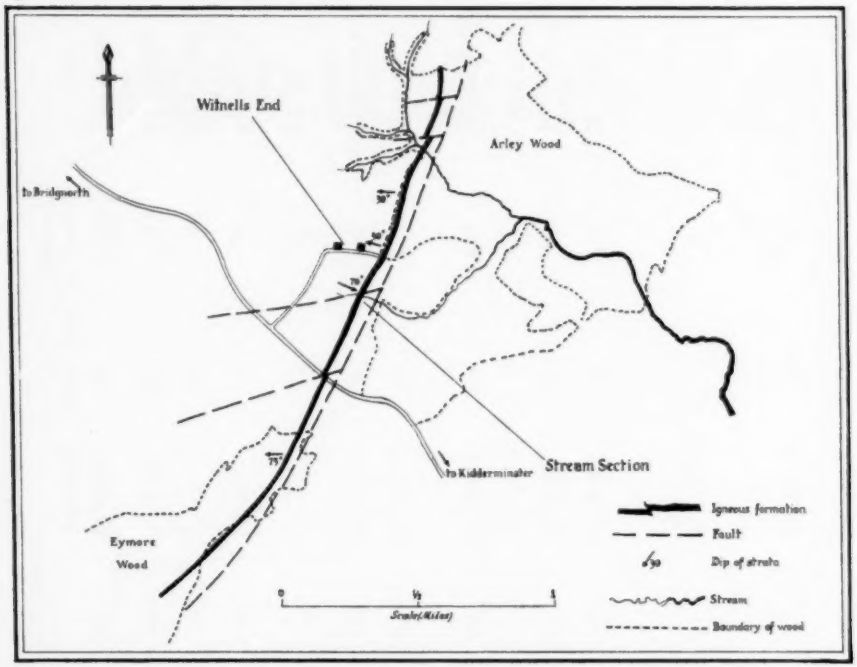

FIG. 3.—The Shatterford formation.

definitely to form part of the same intrusion, this indicates that some of the material has probably undergone self-reversal. However no proof of a self-reversing mechanism has yet been found in laboratory experiments.

There is a discrepancy of about 40° between the mean axis in Figure 4(a) (205° E, 20° up) and the direction for the other Midland basalts (202° E, 17° down). It is interesting to examine the directions relative to the bedding planes. These are given in Figure 4(b), which contains two groups of results for the stream sections, depending on whether the beds are assumed to be inverted (the accepted geological interpretation) or not. On the assumption that the beds are inverted the results are unsatisfactory; but on the other assumption the axes coincide at 200° E and 10° down, in excellent agreement with the results from the other Midland sites.

There are thus two possible explanations of the results. Either intrusion took place after the folding of the sediments, at a different time from the other

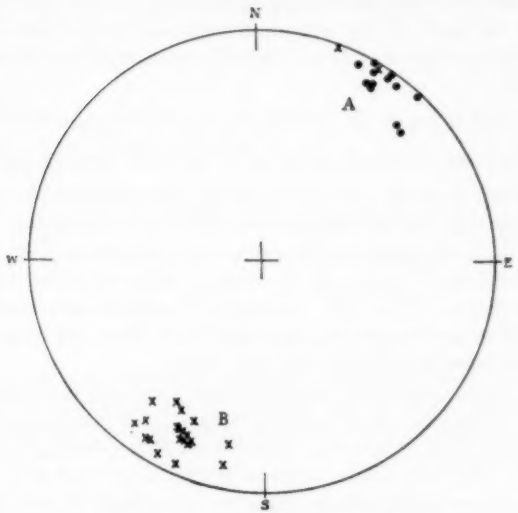

FIG. 4(a).—Results from Shatterford.
Group A: Igneous material at Witnell's End.
Group B: Baked and igneous material from stream section.
● Downward dip; × upward dip; plotted relative to present horizontal.

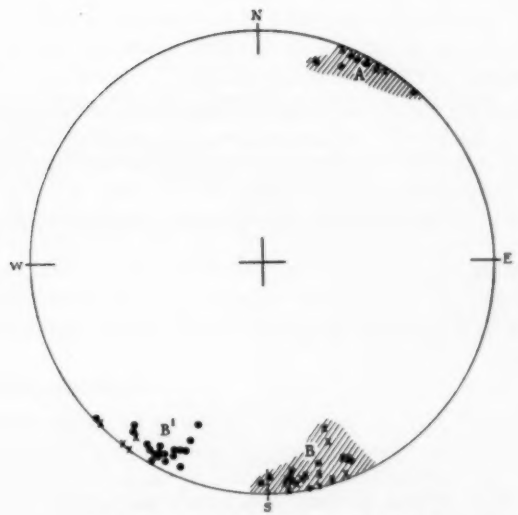

FIG. 4(b).—Results from Shatterford.
Group A: Material at Witnell's End.
Group B: Material from stream section, assuming beds inverted.
Group B¹: As Group B but assuming beds not inverted.
Plotted relative to bedding planes.

formations; or intrusion took place before folding and at the same time as the other formations, but the beds of the stream section are not in fact inverted. A re-examination of the geology may be worthwhile to see whether the second interpretation is possible.

4. Conclusion

The conclusions from the present work may be summarized as follows. The formations at Clee Hill, Kinlet, Shatterford and Pouk Hill appear to be intrusive, whereas Little Wenlock appears to be extrusive. Therefore since Little Wenlock lies in the Carboniferous Limestone, it must be older than the intrusions, which occur in the Millstone Grit. The magnetic directions are consistent with the existence of a difference in age: the mean for Little Wenlock being 355° E and 7° down and for the other sites 202° E and 17° down.

The sites at Barrow Hill and Rowley Regis have not been studied in detail. The geological evidence for their intrusive origin is strong (Marshall 1946). The magnetic directions of the igneous material from these sites agreed well with the directions in the other intrusions, suggesting that they were formed at the same time. The overall mean direction for the intrusions of 200° E and 17° down agrees well with other palaeomagnetic results for the Carboniferous period (Everitt & Belshé, to be published), and thus endorses the conclusions of Pocock & Marshall that the formations were of Carboniferous age.

The measurements from different parts of the formations at Clee Hill and Kinlet suggest that intrusion occurred before the country beds were folded. At Shatterford they indicate that the accepted geological interpretation may require re-examination. Some of the igneous rocks at Shatterford have probably undergone self-reversal.

Acknowledgments

I wish to record my thanks to Dr J. A. Clegg who supervised this work and to Professor P. M. S. Blackett for his continued and stimulating interest in it.

Figure 3 is based on a Crown Copyright Geological Survey map by permission of the Controller of H.M. Stationery Office.

I am indebted to the D.S.I.R. for the award of a research studentship.

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The Palaeomagnetism of some Hypabyssal Intrusive Rocks from South Victoria Land, Antarctica

C. Bull* and E. Irving†

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Summary

The palaeomagnetic pole positions obtained from a study of the directions of magnetization of dolerite sheets and dykes from South Victoria Land lie in what is nowadays the South Pacific.

1. Introduction

The collection of the rock specimens, the palaeomagnetic properties of which are described in this paper, formed part of the work of the Victoria University of Wellington Antarctic Expedition (1958–9) to the Wright Valley area of South Victoria Land, Antarctica. The physical measurements were made at the Australian National University. The Wright Valley is inland from the Wilson Piedmont Glacier on the western shore of the Ross Sea (Figure 1).

The magnetic properties of specimens from two quartz dolerite sheets and from a series of older basic and acid dykes are described. The directions of magnetization of the specimens are all normal (that is, in the same sense as the present Earth's magnetic field) and have declinations to the south-west with inclinations of between 40° and 75°, the inclinations of the older dykes being lower than those of the dolerites.

The age of the dolerite sheet intrusives is uncertain but they have been regarded by some geologists as being approximately contemporaneous with the Mesozoic dolerite intrusives and basalt flows which inject or cover the Gondwana beds in India and the southern continents. The pole position consistent with the direction of magnetization of the dolerite specimens is in the South Pacific Ocean, significantly different from the present pole and from the pole positions obtained from the dolerites and basalts from the other southern continents.

2. Experimental methods

Samples were taken at a number of localities, the geographical co-ordinates of which are given in Table 1. At each locality samples have been taken at one or more sites. In the case of the sheets a site is at a certain level within the sheet; in the case of dykes separate sites refer to individual dykes. One or two separately orientated samples were taken from each site, which were usually cliff faces, and from each sample between one and five disk specimens have been cut. Altogether 79 specimens cut from 37 samples have been studied.

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The intensity (R_n) and direction of magnetization of each disk sample was determined with an astatic magnetometer, using the methods described by Collinson & others (1957). The accuracy of orientation of the specimens is 3°–5°, and the measurement of the direction of magnetization may introduce an additional error of up to 2°. Intensities of magnetization are accurate to about 5 per cent. Declinations (D) are reckoned clockwise from geographic north and inclinations (I) are given relative to the horizontal. It is assumed that the rocks have not been tilted since emplacement. The mean direction of the magnetizations of the specimens obtained from each site is given in Table 1 and the north-seeking directions of the individual specimens are plotted as stereographic projections in Figures 2 and 3. The plane of these projections is the horizontal and GN signifies geographic north. The direction of the present geomagnetic field ($D = 155°$, $I = -83°$) is denoted by F and of the dipole field (0°, $-84°$) by P.

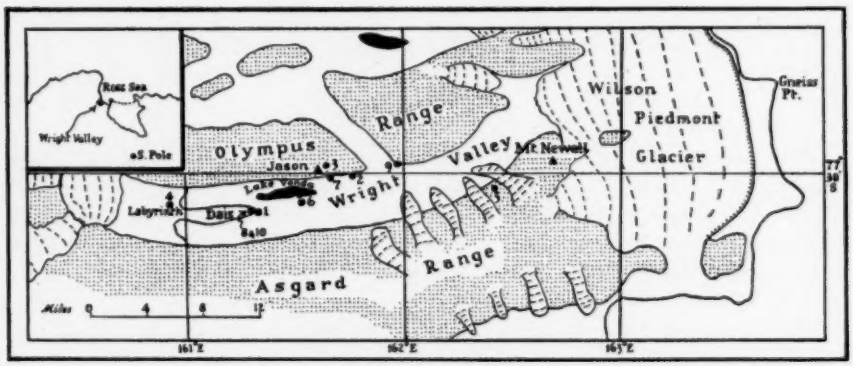

FIG. 1.—Sketch map of the Wright Valley area of South Victoria Land. Glaciers are indicated by dotted lines and land over 2 500 ft by stippling. Localities are indicated by dots, the numbering of which is consistent with that in Table 1.

Specimens from each geological unit have been tested in the laboratory for magnetic stability by subjecting them to alternating magnetic fields at 50 c.p.s. in the absence of a steady field. The specimens were placed in a spinner so that they were demagnetized in all directions by a single treatment, in a method similar to that described by Creer (1959). The direction and intensity of magnetization of the specimens were measured after demagnetization in alternating peak fields of 37, 75, 112, 149, 224, 298, 373, 447, 596 and 745 oersted. These tests indicate whether the primary thermoremanent magnetization, acquired at the time the rock cooled through its Curie point, has been affected by subsequent isothermal magnetization of the softer magnetic components. Such secondary components are likely to have a low coercivity, so that the specimens containing appreciable secondary components will be demagnetized more readily than those containing only the primary magnetization. In addition, if the secondary component is at an angle to the primary one, the direction of magnetization after partial demagnetization will differ from the original direction.

Measurements of the initial susceptibility (σ) of these specimens were made, using an astatic magnetometer, by applying to them a steady field of 0·54 oersted. To determine their saturation remanent magnetization (R_s), specimens were placed

Table 1

Site mean directions

The declination D is reckoned east of geographic north and the inclination I is the angle to the horizontal. R_n is the mean remanent volume intensity e.m.u. per $cm^3 \times 10^{-4}$.

Geological Unit	Locality No.	Geographical co-ordinates	Locality description	Site No.	No. of Samples	No. of Specimen	D deg.	I deg.	R_n
Lower dolerite sheet	1	77° 32′·3 S 161° 19′ E	Dais, alt. 1 500 ft	1a	2	2	246	−74	0·27
			Dais, alt. 1 650 ft	1b	2	2	289	−70	2·72
			Dais, alt. 1 400 ft	1c	1	3	130	−84	0·32
			Dais, alt. 1 700 ft	1d	1	3	182	−76	3·23
			Dais, alt. 1 850 ft	1e	1	1	232	−76	3·59
			Dais, alt. 1 950 ft	1f	1	5	176	−80	1·36
	2	77° 30′·0 S 161° 48′ E	South-east flank of Jason, alt. 1 900 ft	2a	1	2	265	−71	0·96
				2b	2	3	264	−66	0·83
				2c	1	1	247	−62	0·96
Upper dolerite sheet	3	77° 29′·8 S 161° 38′ E	Near summit of Jason, alt. 5 800 ft	3a	1	3	259	−74	12·06
	4	77° 31′·9 S 160° 56′ E	Labyrinth, alt. 3 500 ft	4a	1	3	278	−74	4·08
	5	77° 30′·8 S 162° 25′ E	South side of Wright Valley, near Mt Newall alt. 5 200 ft	5a	1	4	257	−66	0·06

Table 1—*continued*

Geological Unit	Locality No.	Geographical co-ordinates	Locality description	Site No.	No. of Samples	No. of Specimen	*D* deg.	*I* deg.	R_s
Basic dykes in grey granite	6	77° 31′·6 S 161° 28′ E	Middle of Wright Valley, south of Lake Vanda, alt. 470 ft	6a	1	1	222	−57	0·01
				6b	2	5	247	−55	0·02
				6c	1	3	237	−44	0·04
				6d	1	2	237	−40	0·05
				6e	2	4	242	−50	0·03
	7	77° 30′·3 S 161° 40′ E	South flank of Jason, alt. 1 700 ft	7a	1	5	241	−55	0·29
				7b	1	1	244	−52	0·34
				7c	2	3	239	−63	0·01
				7d	2	3	212	−68	0·08
				7e	1	4	231	−70	0·06
Basic dykes in pink granite	8	77° 32′·3 S 161° 18′ E	Dais, alt. 2 100 ft	8a	1	3	311	−76	0·68
				8b	2	5	211	−74	0·16
				8c	1	1	252	−71	0·12
Acid dykes in grey granite	9	77° 29′·2 S 161° 58′ E	East of Jason, alt. 1 400 ft	9a	2	4	280	−50	0·10
Acid dykes in pink granite	10	77° 32·3′ S 161° 18′ E	Dais, alt. 2 100 ft	10a	2	3	303	−64	0·57

in increasing magnetic fields, the remanent intensity being measured at intervals until a constant value was obtained. Incremental magnetic fields in the opposite direction were now applied to the specimens until their remanent magnetization was nullified. The field (H_c) at which this occurs is the coercivity of the saturation remanent magnetization. The values of these properties, and of the intensity of the natural remanent magnetization (R_n), are given in Table 2.

Table 2

Magnetic properties

The following magnetic properties are given for the same samples as are listed in Figure 4: R_n is the volume intensity of the natural remanent magnetization, in e.m.u. per cm^3.; σ is the initial volume susceptibility, that is, the ratio of the intensity of magnetization induced by an external field to the strength of that field, in this case 0·54 oersted; R_s is the saturation remanent magnetization; H_s is the field in oersted required for saturation; H_c is the coercivity in oersted of the saturation remanent magnetization.

Site of specimen	$R_n \times 10^{-4}$	$\sigma \times 10^{-4}$	$R_s \times 10^{-4}$	H_s	H_c
1a	0·29	0·56	48·0	4600	610
5a	0·06	0·25	27·5	3500	650
1f	1·62	10·15	806·0	3500	580
8b	0·18	0·32	>40·0	>7000	1130
6b	0·03	0·18	2·8	5900	1220
9a	0·11	0·14	36·5	4800	740

3. Geology

Topographical and geological surveys of the area were carried out by the Expedition (Webb & McKelvey 1959; McKelvey & Webb 1960). The rock nomenclature used below is that due to Harrington (1958). Throughout the area the basement complex comprises Ross System metamorphosed sediments and Admiralty System intrusives, all probably of pre-Cambrian age. More than 5000 ft of Beacon Group sediments lie non-conformably upon a peneplained surface of the basement. Sheets and sills of Ferrar Dolerites intrude both the basement rocks and the Beacon Sediments. A similar geological succession continues south for at least 700 miles to the Queen Maud Ranges, lat. 86° S (Gould 1935).

In the Wright Valley the Ross System metamorphics, which are confined to the eastern half of the valley, include marble, schist and granulites. The Admiralty System intrusives are exposed as far west as Dais (long. 161° 15′ E) and comprise an older series of porphyritic granites, gneissic granites and pure gneisses (referred to here for convenience as the "grey" granite), intruded by a younger sheet-like body of "pink" granite. Both the Ross metamorphics and the Admiralty granites are intruded by a complex of basic and acidic dykes. Two major dolerite sheets, between 600 and 1500 ft thick, persist through the area. One of these lies wholly within the basement; the other, over most of the area, is intruded at the unconformity between the basement and the overlying Beacon sediments, but at a few localities upward flexures of the sheet expose the Beacon–basement contact. The sills and sheets intruding the Beacon sediments are thinner and do not have as great a lateral extent.

The unconformity between the basement rocks and the Beacon sediments is exposed at 5000ft in the eastern Olympus and Asgard Ranges. It dips at a low angle to the west and at the inland ice plateau edge stands at approximately 3500ft. Similarly the two major dolerite sheets also have a small westerly dip.

The persistent westerly tilt of the sub-Beacon peneplain, of the Beacon sediments and of the later dolerite sheets may be the result of large-scale block faulting. However, the tilt is small, about 3°, and no allowance has been made for it to the n.r.m. vectors given below. There has been no folding of the basement or the overlying Beacon sediments in the area.

Examination of thin sections of specimens from the sheets showed that they are typical quartz dolerites with the exception of the specimen from site 1e which is an acid rock of granophyric aspect. The acid dykes sampled are of aplitic type, and the basic dykes are doleritic with abundant quartz and show signs of deuteric alteration. In all cases the outlines of the ore minerals are clear and sharp and there is no evidence of weathering.

The age of the granites and the dykes is not known but is pre-Mesozoic. The age of the Beacon Sandstones is also uncertain but probably lies within the limits of Upper Palaeozoic and Triassic. The dolerite intrusives are post-Beacon Sandstone and are probably Mesozoic. The value of the palaeomagnetic results suffers from inadequate dating of the formations but it is hoped that this dating can be improved in the near future.

4. The natural remanent magnetization

From the lower dolerite sheet specimens were taken at nine sites in two localities, numbered **1** and **2** in Figure 1. The directions of magnetization of all specimens are plotted in Figure 2. Specimens from five sites have directions of magnetization significantly different from the Earth's field and from the dipole field. Specimens from three sites (1c, 1d, 1f) have directions close to the Earth's field. The direction of the specimens from site 1e was significantly different from the present field, but the magnetization contains an appreciable secondary component. The mean direction of the samples from these four sites is 187°, −81°, and the divergence of 4° from the present field is not significant.

In the upper dolerite sheet one sample was taken from each of localities **3**, **4** and **5** (Figure 1). The directions of specimens from these sites is shown in Figure 2. All of these are significantly different from the present Earth's field.

The directions of magnetization of specimens from ten basic dykes at two localities in the grey granite (localities **6** and **7**, Figure 1) are shown in Figure 3, together with the directions from three basic dykes in the pink granite (locality **8**), and from two acid dykes, one in the grey granite (locality **9**), the other in the pink granite (locality **10**). All these directions differ significantly from the direction of the present Earth's field.

The mean intensity of magnetization at each site is given in Table 1. These vary by three orders of magnitude, the intensity of the dykes generally being an order of magnitude less than that of the dolerites. One sample of grey granite had an intensity of magnetization of less than 10^{-7} e.m.u./cm^3 and the direction could not be measured. Ohe quartzite sample from the Beacon Sandstone had an intensity of $2{\cdot}62 \cdot 10^{-6}$ and an inclination relative to the bedding (which is horizontal) of 64° which is much less than the present Earth's field.

5. Magnetic stability

Apart from those of specimens at the four sites in locality 1, the individual directions of magnetization all diverge from the present Earth's field. Table 3 gives the mean direction of the dolerite sheets and of the basic dykes in the grey granite. The divergence of these directions from the present field and from the dipole fields exceeds the error ($P = 0{\cdot}05$) by a factor of three or more. This deviation test is evidence for the stability of the directions of magnetization.

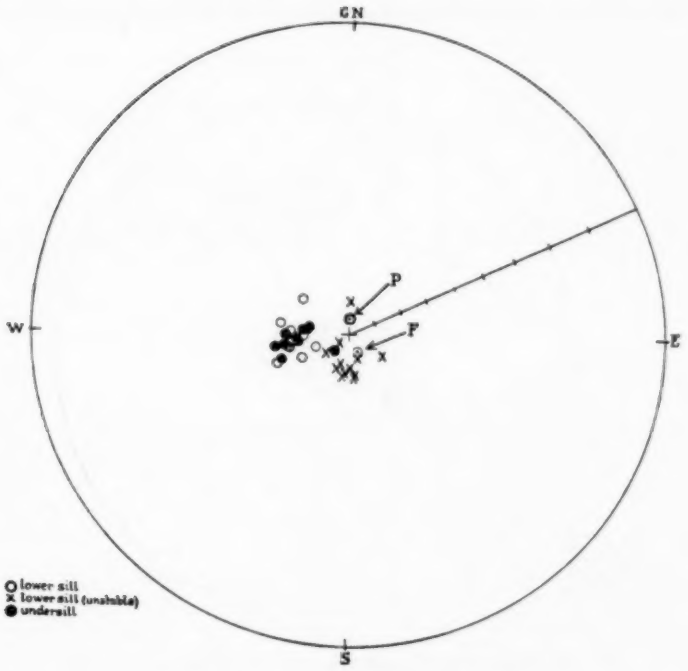

FIG. 2.—Directions of magnetization in the dolerite sills. Specimens from the lower sill are indicated by circles (stable specimens) and by crosses (direction along the present Earth's field and unstable specimen 1e). Specimens from the upper sill are indicated by dots. A stereographic projection is used in this figure and in Figures 3 and 5, and the directions are plotted on the upper hemisphere.

Alternating field demagnetization curves have been obtained for 13 specimens. An example from each geological unit is given in Figure 4. In all cases, except those for the specimens with direction along the Earth's field, and the specimen from site 1e, the intensity of magnetization was not reduced by more than one-quarter by treatment in an alternating field of 150 oersted. In the remaining specimens (e.g. Figure 4 (1f)) the intensity of magnetization was reduced by a factor of three in the same field. It may be noted that with the specimens of curves 1a and 8b the intensity of magnetization increases after demagnetization in small fields, while the direction remains unchanged. The reason for this increase is obscure but it is consistent with the presence of a subordinate, low-coercivity

component of magnetization opposed in direction to that of the dominant component.

The associated changes of the directions of magnetization after partial demagnetization are shown in Figure 5. Where the initial direction of magnetization diverged from the present field the directions of the magnetization remaining after treatment in fields of up to 370 oersted were little changed, except with the specimen from site 1e. In some cases (e.g. specimen from site 1a) the direction remained unchanged in fields of 750 oersted. However, the directions of magnetization of those specimens whose initial direction was close to the present field and of the one from site 1e, changed significantly in fields of less than 200 oersted.

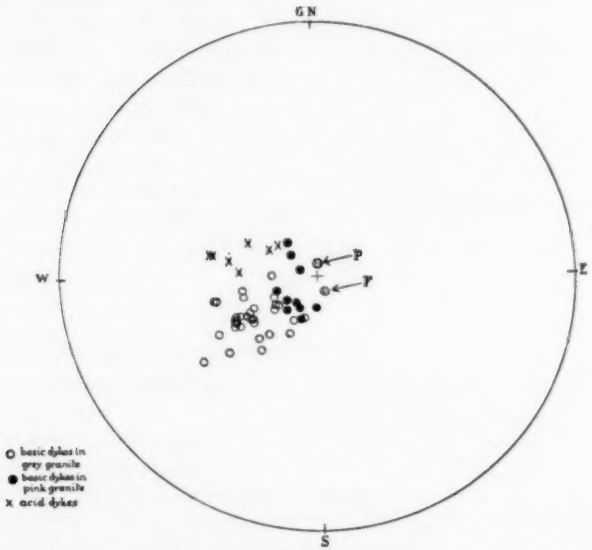

FIG. 3.—Directions of magnetization in the basement dykes. Specimens from 10 basic dykes in the grey granite are indicated by circles; specimens from 3 basic dykes in the pink granite by dots and from two acid dykes by crosses.

6. Analysis

From the evidence discussed above it may be supposed that with the exception of samples from sites 1c, 1d, 1e and 1f, the specimens from these intrusions acquired their direction of magnetization parallel to the direction of the Earth's magnetic field existing at the time they cooled through their Curie points. The specimens from sites 1c to 1f very probably have a secondary magnetization along the present Earth's field, and provide no information about the ancient field. They are not considered further.

The two dolerite sheets were intruded at different times and for each sheet it is likely that the time taken for the Curie point to pass from the margin to the centre was about 500 years (Jaeger 1957). Thus the direction of magnetization of the stable samples from 8 sites, which span 250 ft in the lower sill and 600 ft in

the upper sill, may have averaged out the secular variation. Similarly the directions of the magnetization of the 10 basic dykes in the grey granite are likely to have recorded the direction of the field over a considerable period, since the dyke swarm sampled at locality **7** cuts the dykes sampled at locality **6**. Assuming that the secular variation is fully represented in the scatter of directions observed, and that the main Earth's field was a geocentric dipole, the ancient pole positions appropriate to the periods when these sills and dykes cooled through their Curie

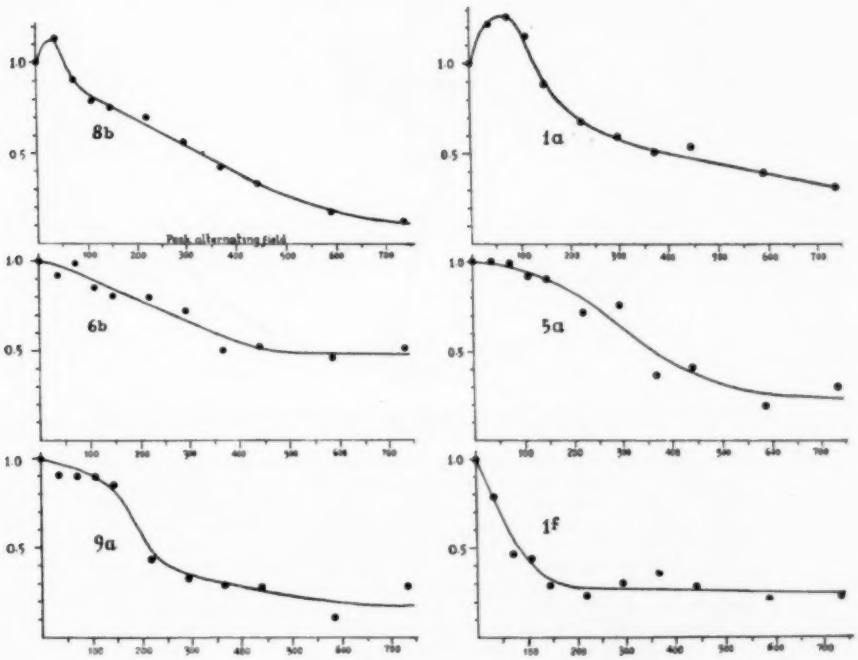

FIG. 4.—Alternating field demagnetization curves.

(1a) Lower dolerite (stable); (5a) upper dolerite; (1f) lower dolerite (initial magnetization parallel to the present Earth's field); (8b) basic dyke in pink granite; (6b) basic dyke in grey granite; (9a) acid dyke in grey granite. The numbers refer to the sites from which the specimens were taken. The ordinate gives the ratio of the intensity of magnetization remaining after treatment in the peak alternating field in oersted recorded along the abscissa, to the natural remanent magnetization, R_n.

points may be calculated from their mean directions of magnetization. These mean directions, which are given in Table 3 have been computed as a mean of the directions at the 8 sites in the sheets and from 10 basic dykes, giving each site unit weight. The directions at each site are themselves means of several observations (see Table 1). It is noteworthy that the scatter of directions of magnetization between sites in the sheets (standard deviation 10°) is comparable with that obtained in the Tasmanian dolerites (9°) (Irving 1956).

A similar analysis has not been made of the results from the basic dykes in the pink granite or from the acid dykes because the data are insufficient.

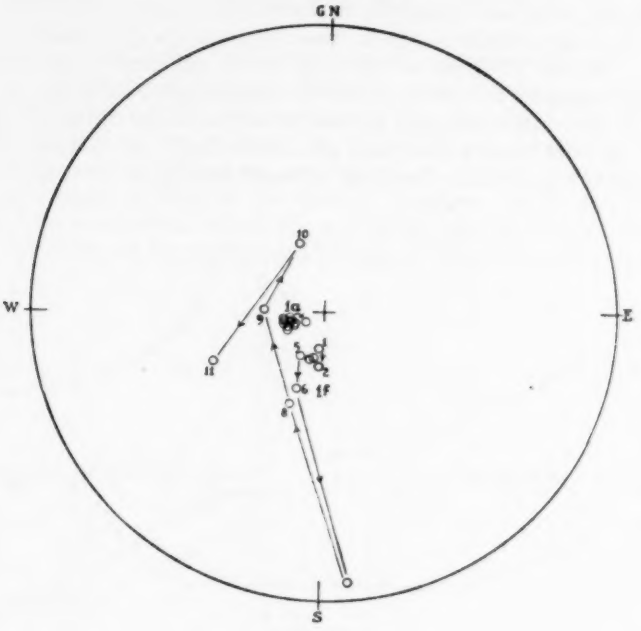

FIG. 5a

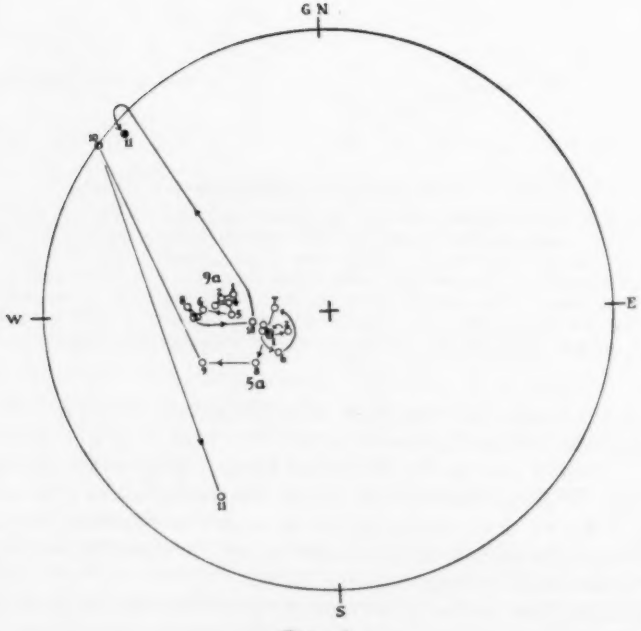

FIG. 5b

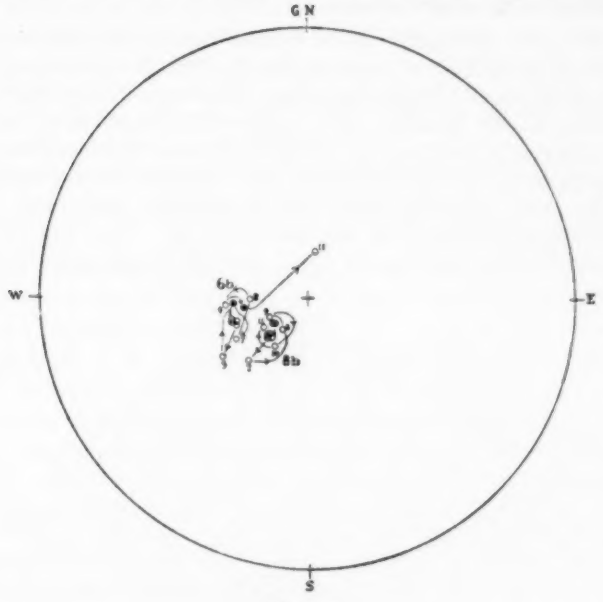

FIG. 5c

FIG. 5.—Changes of direction of remanent magnetization produced by treatment in alternating magnetic fields.

The three stereograms in this figure give the directions observed in the six specimens whose demagnetization curves are shown in Figure 4. The numbering is as in Figure 4. Points labelled 1 are the directions of the natural remanent magnetization. Points labelled 2–11 are the directions after treatment in alternating fields of 37, 75, 112, 149, 224, 298, 373, 447, 596 and 745 oersted respectively. Where possible the directions after successive increasing demagnetizations are linked, but in some cases the changes in direction are so small that this is not possible without confusion. For the same reason it has not been possible to number all points.

Table 3

Mean directions of magnetization and pole positions

N denotes the number of sites, the individual directions of which are given in Table 1. *D* and *I* are the declination and inclination of the resultant direction of magnetization computed by giving each site unit weight. *R* is the length of the resultant. α is the semi-angle of the cone of confidence at $P = 0{\cdot}05$ (Fisher 1953). ΔF and ΔP are the divergences of the mean direction from the present Earth's field (155°, −83°) and from the dipole field (0°, −84°) respectively. The southern hemisphere pole positions are specified by latitude and longitude and the errors $d\phi$ and $d\chi$ are in the direction of and at right angles to the co-latitude, respectively.

Geological unit	*N*	Mean direction of magnetization				Pole position		$d\phi$	$d\chi$	Divergences	
		D	*I*	*R*	α	Lat.	Long.			ΔF	ΔP
Dolerite sills	8	262	−70	7·889	7	51° S	132° W	10	12	23	21
Basic dykes in grey granite	10	237	−56	9·832	7	29° S	149° W	7	10	32	38

7. Comparison with other results

Pole positions for the dolerite sheets and the basic dykes are plotted in Figure 6. The pole for the dykes is in a lower latitude than that for the sheets. The dykes pre-date the sheets, so that this change of pole position is in accord with the results from Australia, North America and Europe where, with few exceptions, the divergence of the palaeomagnetic pole from the present pole increases with time, at least for periods back to the early Palaeozoic. Since these results were obtained, pole positions from other Antarctic dolerite intrusion have been reported by Turnbull (1959) and Blundell & Stephenson (1959). They are 58° S, 142° W, 54° S and 136° W respectively, and agree very well with our result from the dolerite sills.

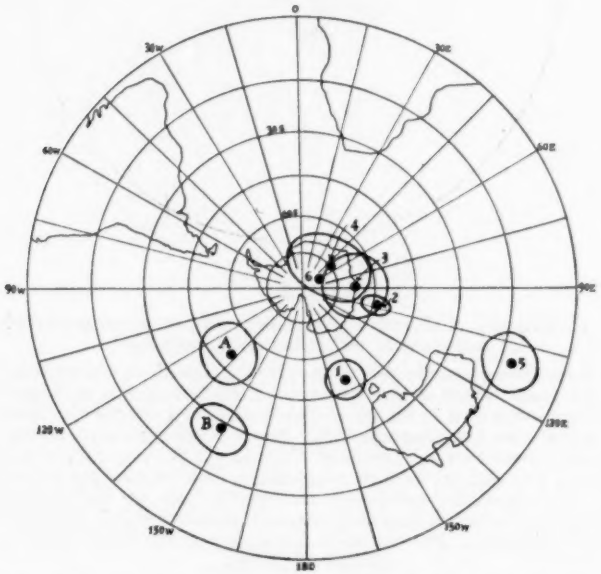

FIG. 6.—Pole positions relative to the present distribution of continents. The pole positions are numbered as follows: (A) dolerite sills of Wright Valley; (B) basic dykes of Wright Valley; (1) Tasmanian dolerites (Irving 1956); (2) Karroo basalts of Rhodesia (Nairn 1955); (3) the average pole from the dolerite sills and baked contact rocks in the mines of Estcourt and Winkelhaak, South Africa (Graham & Hales 1957, see Irving 1959); (4) Karroo dolerites, surface samples, South Africa (Graham & Hales 1957); (5) Rajmahal traps of Bihar, India (Clegg & others 1958); (6) Serra Geral lavas and baked Botacatu Sandstones of Uruguay (Creer 1958). The map is a stereographic projection.

The pole positions from dolerites, basalts and associated baked sediments from India and the southern continents, relative to the present distribution of continents, are also shown in Figure 6, the formations being listed in the legend. The poles given by the African and South American rocks do not differ greatly from each other, but both diverge appreciably from those given by the Indian rocks and the Tasmanian and Antarctic dolerites, which themselves are not consistent.

The age relationships of these dolerites and basalts are not well known. In most cases they intrude or overlie Triassic beds, which fix their lower limit. Their

upper limit, however, is less satisfactorily determined. In some cases, like the Rajmahal traps, interbedded fossil-bearing sediments give the age as almost certainly Jurassic, while in others such as the Tasmanian dolerites, the upper limit could extend into the Lower Tertiary. The age of the Antarctic dolerites is even less well determined. However, it seems probable that all of these rock formations date from the Jurassic or later Mesozoic.

The divergence of the pole positions could be due to several causes:

(a) relative movement of the continents since the time of formation of these rocks;

(b) polar wandering during the time spanned by these rock formations;

(c) the Earth's magnetic field not being a dipole at the time that these rocks acquired their magnetization; and

(d) the invalidity of the correlation of the present directions of magnetization with the field at the time at which the rocks cooled.

At present we cannot decide with certainty between these; in fact, all may be responsible in part for the divergences observed. However, in connexion with the first possibility it is of interest to test the extent to which these pole positions are consistent with reconstructions of the past distribution of the continents derived from other evidence. Clearly, if our procedures and the reconstruction are correct then pole positions, previously divergent, should unify.

Many such reconstructions are available, but the one chosen here is that due to Du Toit (1937), because it was the first reconstruction accompanied by a good map. The pole positions calculated in accordance with du Toit's distribution of the Gondwanaland continents in the late Palaeozoic are given in Figure 7. The poles from India, South America and Tasmania are close together, while those from Antarctica and Africa are some distance away. With this distribution of

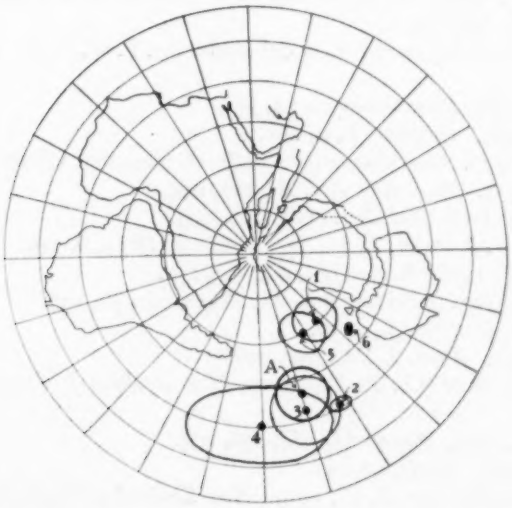

FIG. 7.—Pole positions relative to du Toit's reconstruction of Gondwanaland. The poles are labelled as in Figure 6. The continental outlines are taken from du Toit (1937), Figure 7, p. 58, who used an equal-area projection.

continents, the average angular separation of the poles, taken two at a time, is 24° and the dispersion, calculated by the method of Fisher (1953) is given by $k = 25$. The corresponding figures for the pole positions relative to the present distribution of the continents (Figure 6) are 52° and $k = 4{\cdot}6$ so that du Toit's reconstruction substantially improves the fit of these pole positions. The divergences remaining in Figure 7 could be caused by the rock formations in the various continents not being exactly contemporaneous. It must also be noted that du Toit's reconstruction refers to the Palaeozoic while the formations here discussed are Mesozoic and it is possible that the break-up of Gondwanaland had already begun at the time of their formation.

8. Acknowledgments

It is a pleasure to express our thanks to the organizers and members of United States Operation Deep Freeze IV, to the New Zealand Ross Dependency Research Committee, to the University of New Zealand and to the many individuals and commercial firms whose encouragement and support made the Expedition possible.

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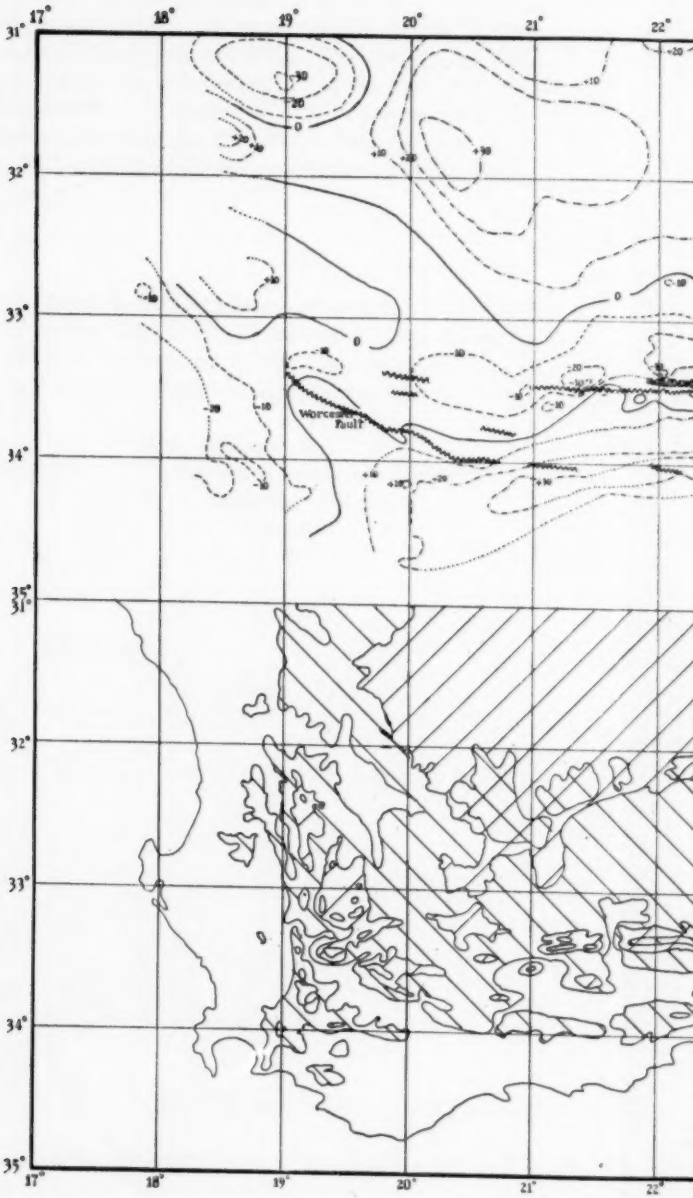

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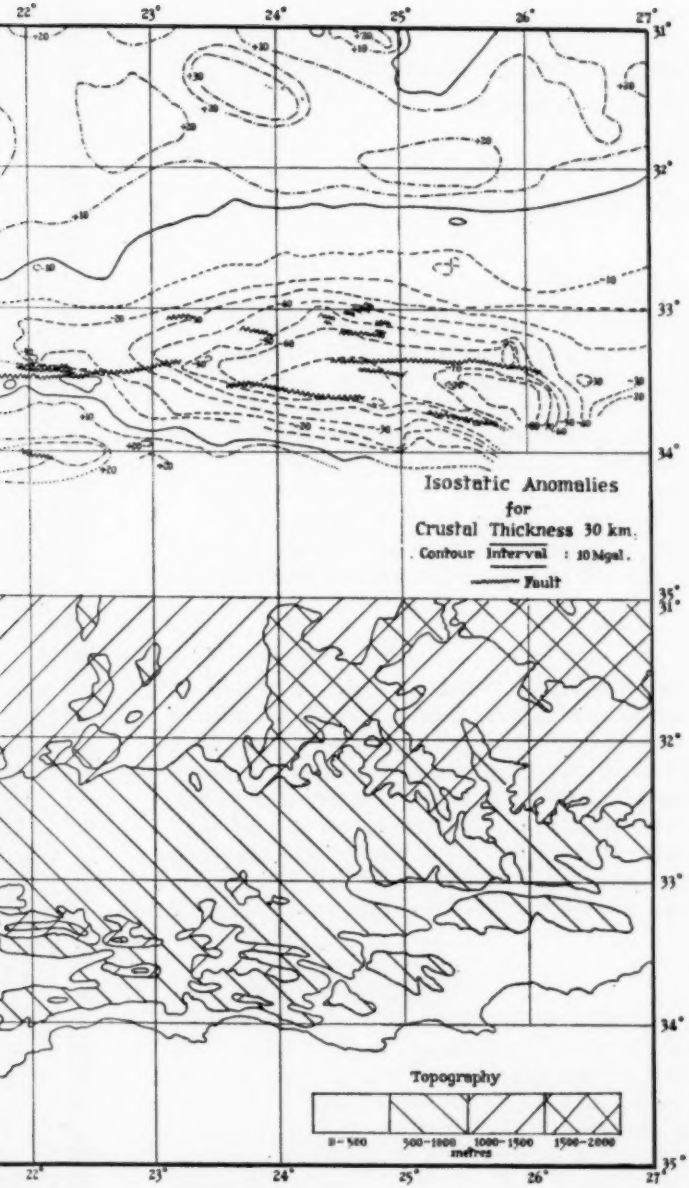
FIG. 1.—Isostatic anomalies and topo



nd topography in the Southern Cape.

Isostatic Anomalies and Crustal Structure in the Southern Cape

A. L. Hales and D. I. Gough

(Received 1959 November 21)

Summary

It has been found that there is a general tendency for the isostatic anomalies to be negative in the region between the escarpment and the sea. The authors consider that these negative anomalies arise from the compensation of topography which has been removed by erosion. The paper discusses the negative anomaly in the Southern Cape. It is found that at least 1·4 km has been removed over a strip 80 km wide. The stress differences which result are of the order of $2{\cdot}3 \times 10^8$ dyn/cm^2 while the load is being carried by both crust and mantle. If the crust is floating on the mantle the stresses in the crust are of the order of 120×10^8 dyn/cm^2 in the centre of the strip.

1. Introduction

In a recent paper on gravity anomalies in relation to crustal thickness it was pointed out that the isostatic anomalies were systematically negative on the seaward side of the great escarpment which lies at a distance of about 200 km from the eastern and southern coasts of South Africa (Hales & Gough 1959). In these areas it is known that extensive erosion has been going on since Cretaceous time, or earlier. In the paper quoted it was suggested that the negative isostatic anomalies could be explained on the hypothesis that the roots of the eroded topography still existed. It is the purpose of this paper to examine in detail one of the negative anomaly zones, namely that in the Southern Cape.

2. The isostatic anomalies

Figure 1 shows the isostatic anomalies on the Airy–Heiskanen hypothesis with sea-level crustal thickness 30 km, for the area bounded by 17° and 27° E, 31° and 35° S, together with the topography on the same scale and projection. It is easy to see that negative anomaly is closely associated with the eroded area below the great escarpment. In the neighbourhood of 25¾° E, 33½° S, the anomaly falls to below −80 mgal. In this region, however, the geological map shows Cretaceous sediments which may be of lower density than the value 2·67 g/cm^3 which was assumed for the crust in computing the isostatic anomalies. For this reason the mass deficiency producing the anomaly will be estimated for sections along the meridians at 24½° E and 25¼° E. On the meridian 24½° E the surface rocks are quartzites and shales of the Cape system with densities not very different from the

assumed value of 2·67 g/cm³. On the meridian 25¼° E there is a narrow strip of Cretaceous sediments near the centre line of the anomaly. The gravity profiles along these meridians are plotted in Figures 2 and 3, together with the corresponding topographic profiles. It will be noted that in each case there is a positive anomaly belt at the inland, or northern end of the profile. There is also a suggestion of a similar positive belt on the seaward side of the negative anomaly region.

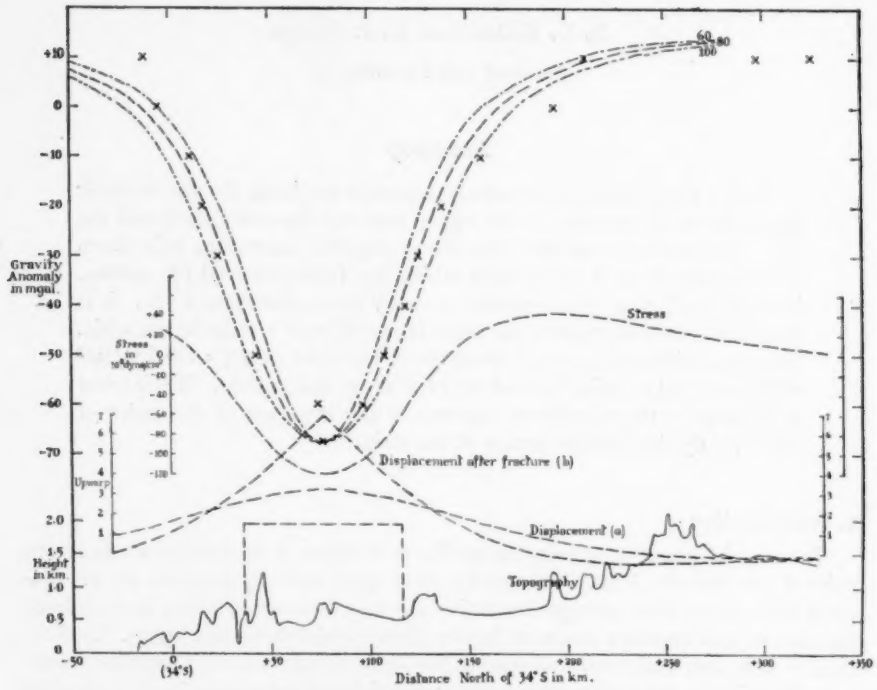


FIG. 2.—Gravity profile along 24¼° E.

We have suggested that the negative anomaly represents the gravity effect of the compensation of the topography removed by erosion. Whether or not this is the case, it is clear that the crust in this region must experience a very considerable upthrust. It is possible that the belts of positive anomaly are indicative of a more or less elastic upwarping of the crust caused by this upthrust. In the case of the seaward belt of positive anomaly off shore, deposition of sediment will no doubt have contributed to the anomaly. No such simple alternative explanation can be found for the inland belt. In this connection it may be remembered that Jeffreys (1932) has suggested that the development of such features takes place in three stages. "In the first so long as the disturbance is not too great the stress differences in the asthenosphere will not exceed its strength and Darwin's theory (or the modifications of it dealt with above) will apply. In the second the disturbance becomes great enough to produce yield in the asthenosphere, and the stress differences in the lithosphere are accordingly increased to those appropriate in a floating crust. In the third the stress differences in the crust itself come to

exceed its strength, and yield in the crust keeps pace with deposition or denudation so that $g\sigma a$ never exceeds the strength of the material."

The existence of large belts of negative anomaly which must have existed for geologically considerable periods of time is of importance in relation to the question of the finite viscosity in the mantle. Jeffreys (1952a) remarks that "the nearest approach to positive evidence for finite viscosity in the lower layer is that provided by the Fenno–Scandian uplift". He goes on to say that if this is to be generalized to the Earth as a whole, then it invites further check. "If it is true in general, it will follow that every region of positive gravity anomalies is sinking

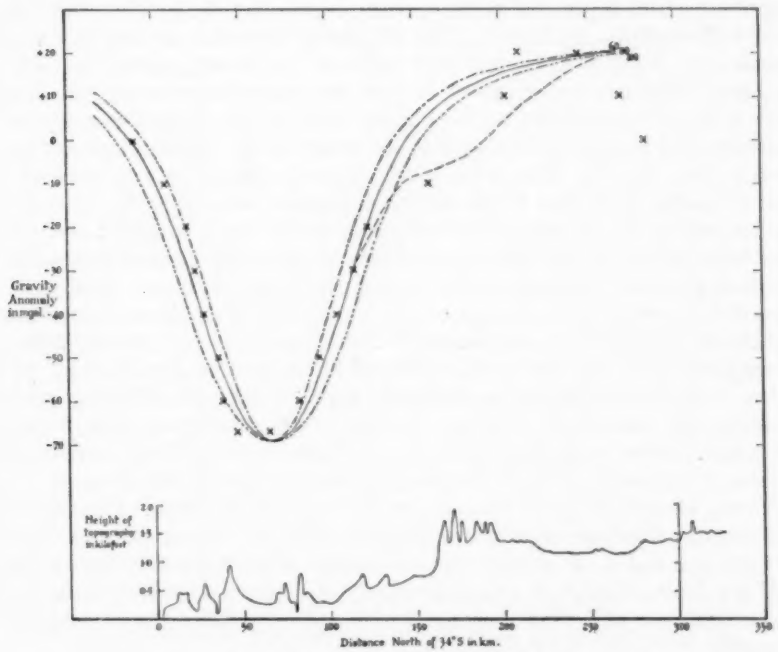


FIG. 3.—Gravity profile along 25¼° E.

and every region of negative ones rising. In any mountain system undergoing denudation, either compensation should keep pace with denudation and there would be no systematic isostatic gravity anomalies, or it would fail to keep pace and the gravity anomalies would be negative. Where loads have been added recently there should be either no gravity anomalies or systematically positive ones. At every point these consequences are contrary to the facts." Jeffreys points out, moreover, that there are positive anomalies in the Welsh mountains and in the Highlands of Scotland.

There is general agreement that the erosion in South Africa is of long standing. King (1951) remarks that the great escarpment is "an erosion scarp throughout and is tolerably ancient, the first traces having appeared soon after the close of Karroo sedimentation in early or middle Jurassic, and by the middle Cretaceous it was already a well-marked feature". Hamilton & Cooke (1954) remark that the Southern Cape coast has been uplifted since mid-Cretaceous. The last-named

authors state that the coastal regions of South Africa are believed to be in process of an elevation today.

3. Estimates of the deficiency of mass

It is clear from Figure 1 that it is reasonable to treat the anomaly at each of the chosen section lines as being of infinite length in the direction at right angles to the section. In order to estimate the deficiencies of mass responsible for the gravity anomalies shown in Figures 2 and 3, theoretical anomaly curves have been computed for mass deficiencies concentrated on parallel-sided plane strips of infinite length at various depths below the surface. In this way the problem was reduced to two dimensions. In Figure 4, the calculated anomalies are due to mass deficiencies at a depth of 35 km and with widths of 40, 80 and 120 km. In each case the mass deficiency was adjusted so that the maximum anomaly was 84 mgal. This is the difference between the highest value of the positive anomaly on the northern flank and the lowest value in the trough of the negative anomaly for the section along 24½° E. For 40 km width the mass deficiency corresponded to a root of thickness 10·1 km if the density difference was 0·6 g/cm³. For 80 and 120 km widths the corresponding thicknesses of the "root" were 6·2 and 5·0 km. The other curves in this figure were drawn to show the effect of increasing the depth of the mass deficiency while keeping the width constant. Once again the mass deficiency has been adjusted to give an anomaly of maximum value 84 mgal. Deficiencies equivalent to 5·2, 6·2 and 7·2 km of material with a density difference of 0·6 g/cm³ were required over a width of 80 km at mean depths of 25, 35 and 45 km. It will be noted that increasing the depth of the mass deficiency produces somewhat the same kind of change of shape of the anomaly curve as increasing the width. After comparing the curves of Figure 4 with the observed gravity profiles of Figures 2 and 3, it was decided that the best fit was given by a mass deficiency of width between 60 and 100 km at a depth of 35 km. The theoretical anomalies for distributions at this depth with widths 60, 80 and 100 km are plotted on Figures 2 and 3. It is clear that an estimate of the mass deficiency as 80 km wide at a depth of 35 km fits the observations of Figure 2 reasonably well. In this case a thickness of 6·2 km of material with a density difference of 0·6 g/cm³ is required.

It should be noted that if the mean depth of the mass anomaly be changed to 25 km then the width of the deficiency must be increased to about 97 km. The total mass deficiency is changed by only about 5 per cent. Similarly, if the mean depth be changed to 45 km, the width of the mass deficiency required is decreased to about 64 km. The total mass deficiency is changed by about 11 per cent. Uncertainty regarding the mean depth of the deficiency does not therefore affect the width of the mass deficiency very greatly, and the effect on the total amount of the mass deficiency is even smaller. It will be seen from the same data that the error introduced by concentrating the mass deficiency in a plane is small compared with the other uncertainties involved.

A similar comparison was made for the section along 25¼° E. It is clear that a mass deficiency of width 80 km and depth 35 km fits the observations reasonably well. The mass deficiency required is equivalent to 6·8 km of material with density difference 0·6 g/cm³.

It will be noted that the greatest departure of the theoretical curves from the observed values is on the northern flank. An additional root of thickness 2·7 km

and width 20 km at mean depth 35 km, results in the theoretical anomaly curve shown dashed in Figure 3. The fit has been very greatly improved although the total mass deficiency has been changed by only 9 per cent. The gradients, however, suggest that the deficiency responsible is nearer the surface than 35 km. In that case the mass deficiency required is less than the 9 per cent estimated.

In considering these estimates of the mass deficiency which would produce the observed anomalies it should be remembered that there is no unique distribution of mass corresponding to a given gravity anomaly. In the present case, however, it is known that the surface rocks have densities in the neighbourhood

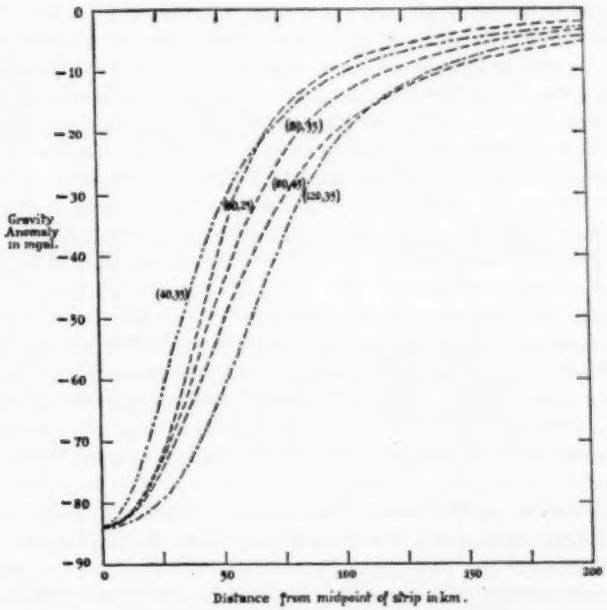


FIG. 4.—Calculated gravity anomalies for strips, 40, 80 and 120 km wide at a depth of 35 km, and for a strip 80 km wide at depths of 25 and 45 km.

of 2·7 g/cm³. The known geology would seem to preclude a density deficiency near the surface as the cause of the negative anomaly. Under these circumstances the mass deficiency must have an overall magnitude at least as large as that estimated above. However, the mass deficiency may have been much greater, for the maximum value of the anomaly before deformation of the crust may have been greater than the estimate of 84 mgal used in making the calculations. This possibility will be discussed further in Section 5.

It has been suggested that the negative anomaly arises from the compensation of topography now removed by erosion. Assuming a density of 2·67 g/cm³ the thickness of rock removed would be 1·38 km for 80 km along 24¼° E and 1·53 km for 80 km along 25¼° E. The mean heights of the present topography along these 80 km strips are 0·62 and 0·41 km respectively. The dashed lines on the topographic sections of Figures 2 and 3 indicate how much material has been removed. It should be noted that even if the original topography deviated considerably

from the simplified version shown in the figures, the estimates of the total amount of material removed would be changed very little by such redistribution.

4. The stresses in an elastic crust

In order to calculate the stress distribution in the crust and mantle arising as a result of the erosion it is necessary to make some assumption with regard to the initial stress-distribution. The assumption made here is that prior to the erosion the stress distribution was the hydrostatic one. We assume further that the curvature of the Earth can be neglected and the problem reduced to one of finding the stresses in a semi-infinite medium bounded by a plane and with a uniform two-dimensional load $-\sigma$ per unit area over a strip of width l. For this distribution it can be shown (Timoshenko 1934) that the difference of the principal stresses at a point P is equal to $2g\sigma \sin\alpha/\pi$ where α is the angle subtended at P by the end points of the strip. It is clear that the stress difference is constant along all circles passing through the end points of the loaded strip and further that the stress difference will be a maximum when $\sin\alpha = 1$, i.e. on the semicircle having the strip as diameter. The value of the maximum stress difference is $2g\sigma/\pi$. For the case with which we are dealing the width of the loaded zone is 80 km and the greatest depth at which the maximum stress difference occurs is 40 km. The maximum stress difference is equal to $2{\cdot}3 \times 10^8$ dyn/cm^2. At depths of 80 and 120 km the maximum stress differences are $1{\cdot}8$ and $1{\cdot}4 \times 10^8$ dyn/cm^2 respectively. Thus although the maximum stress difference occurs just below the crust the stress differences are of the same order to considerable depths below the surface. In our view some upwarping of the crust has taken place. This implies that failure has taken place in the mantle so that one may conclude that the strength of the mantle is not greater than $2{\cdot}3 \times 10^8$ dyn/cm^2. It may of course be less, for there is no means of establishing at what time, or at what depth, the failure in the mantle began.

The displacement of the plane surface of a semi-infinite body is not determinate unless some assumption is made with regard to the displacement at depth. The assumption made by Timoshenko is that at some depth d, the displacement is zero. On this assumption it is possible to calculate the shape of the free surface. Taking the displacement at the centre of the loaded zone to be zero, the displacements given in Table 1 are found.

Table 1

Distance from midpoint of strip (in km)	0	20	40	120	200	440	840
Displacement (in cm)	0	240	1280	3840	4800	6270	7460

These displacements are small in comparison with those which occur in the second stage as will be seen in the next section. The need to make the arbitrary assumption of zero displacement on a plane at depth d suggests that the use of the theory for a semi-infinite solid may not be warranted when calculating the displacements at the surface of a sphere.

5. The stresses in a floating crust

In the second stage it is assumed that the mantle is devoid of strength and that the crust, which is elastic, floats on the mantle material. We assume that there

is a uniform load σ over a strip of width $2l$. The displacement of the crust, w, is given by (Jeffreys 1952b):

$$D\frac{d^4w}{dx^4} = g\sigma - g\rho' w \quad \text{for } -l < x < l,$$

$$= -g\rho' w \qquad \text{for } x > l \text{ and } < -l,$$

where

$$D = \frac{1}{3}\frac{\mu(\lambda+\mu)}{\lambda+2\mu}H^3,$$

H being the thickness of the crust and ρ' = density difference between the crust and the mantle.

From these equations it follows that

$$w = \frac{\sigma}{2\rho'}\{2 - e^{-\beta l}\cos\beta l(e^{\beta x}+e^{-\beta x})\cos\beta x - e^{-\beta l}\sin\beta l(e^{\beta x}-e^{-\beta x})\sin\beta x\} \qquad \text{for } -l < x < l$$

and

$$w = \frac{\sigma}{2\rho'}e^{-\beta x}\{(e^{\beta l}-e^{-\beta l})\cos\beta l\cos\beta x + (e^{\beta l}+e^{-\beta l})\sin\beta l\sin\beta x\} \qquad \text{for } x > l,$$

β^4 being equal to $g\rho'/4D$.

The values of w for $l = 40$ km and a root thickness of 6·2 km are shown in Figure 2. The maximum upwarp is 3·2 km at the midpoint of the strip. Values of the stress p_{11} have also been calculated. The greatest value of p_{11} is at the midpoint and is equal to 120×10^8 dyn/cm^2. This is more than 30 times the maximum load. The other maximum (numerical) values of p_{11} occur at distances of 125 km from the midpoint of the strip and are equal to 42×10^8 dyn/cm^2, i.e. approximately one-third of the value in the middle of the strip. It would be expected therefore that failure would occur first on the centre line of the anomaly.

If failure occurred along this line, and in consequence the condition that the crust should remain horizontal along the centre line were removed, the first effect would be that there would be a further rise along this line. There would not, however, be any tendency for differential movement between the sides of the fault. The shape of the crust after fracture along the centre line is plotted in Figure 2. It should be noted that for the upwarp to change in shape from (a) to (b) it is necessary that there should be failure, or at any rate movement, in the mantle. This means that the process of change from (a) to (b) would be a gradual one. The points of maximum stress are now between 50 and 60 km from the centre line and eventually failure will occur at these points. As a result of this last failure the root block will rise and the negative anomaly will tend to disappear. The final stage would then be the disappearance of the upwarping as a result of new movements in the mantle.

The whole of the preceding discussion is based upon the premise that the maximum value of the negative anomaly is 84 mgal. This is a minimum estimate for it has been assumed that the zero line from which the negative anomaly should be measured is that given by the greatest value of the positive anomaly on the northern flank. Having calculated w, the upward displacement arising from elastic deformation of a floating crust, it is possible to calculate the positive gravity anomaly which would result from the upwarping when complete. This can be combined with the

negative anomaly and the resulting curves fitted to the observations. The results of this operation are shown in Figures 5 and 6 for the profiles along 24½° E and 25¼° E respectively. It will be seen that a reasonably good fit is obtained with a deficiency 120 km wide at depth 35 km. The mass deficiency required would correspond to the removal of 2·8 km of topography over the 120 km strip, i.e. rather more than twice as much as was estimated in Section 3. The new estimate gives a maximum value in contrast with the minimum estimate of Section 3. The upwarping and stress differences would also be increased by a factor of about two. The maximum values of the upwarping and stress will only be attained if the crust

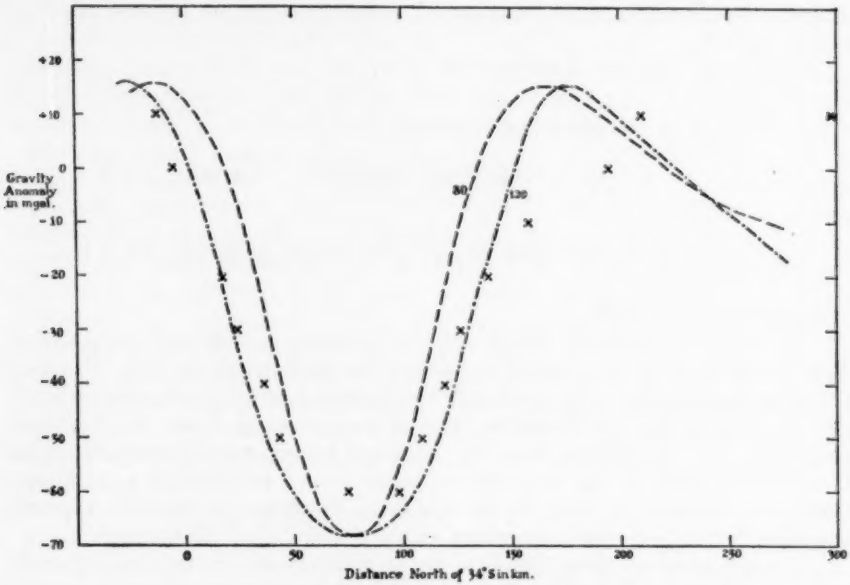


FIG. 5.—Comparison of the calculated gravity anomaly profiles allowing for the effect of upwarping with observations along 24½°E.

is sufficiently strong to resist fracture until failure in the mantle region is complete. The high stresses in the crust which are of the order of 240×10^8 dyn/cm^2 make this improbable.

Under the circumstances outlined above the region near an actively eroding area undergoes a complicated series of vertical movements of quite considerable magnitude.

6. Field observations

There is general agreement that there have been extensive warping movements around the coast of Africa from Jurassic times onwards. Details of these movements are given by du Toit (1954), Dixey (1955), Gevers (1942) and King (1951). If the upwarping followed the pattern outlined earlier in this paper it is reasonable to expect that failure would occur in adjacent blocks at different times, the times of failure being set in part by the rate of erosion.

The faults in the area under discussion have been marked on the gravity map. All the major faults are shown in the area to the east of 20° E, but west of that meridian only the Worcester fault is shown.

It will be noted that in the area east of 22° E where the gravity low occurs there is a fault system which runs not very far from the centre line of the anomaly as would be expected from the discussion in Section 5. So far as can be ascertained none of these faults has a very large throw. This is in accordance with the type of failure to be expected along the centre of the anomaly.

In the area between the sections along 24½° E and 25¼° E Cretaceous beds occur at an elevation of 0·64 km not far from the centre line of the anomaly. These beds are not marine but there is a marine facies higher in the system and it follows therefore that the minimum uplift is 0·64 km. On the assumption that this uplift

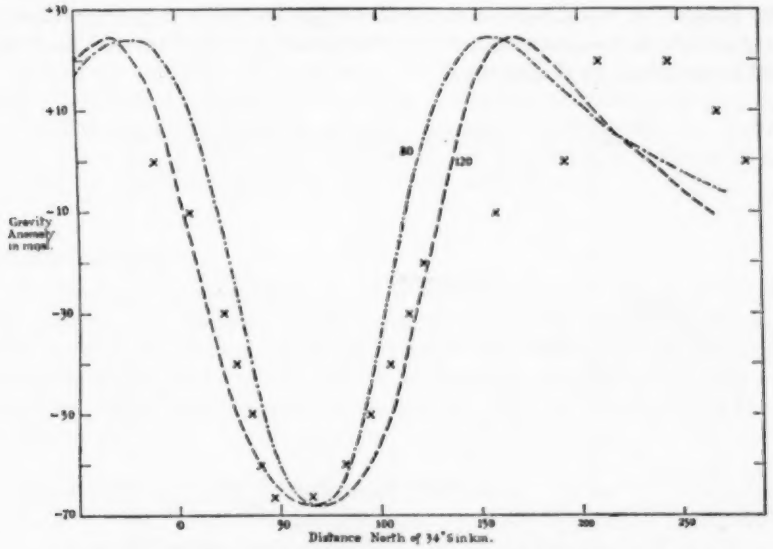


FIG. 6.—Comparison of calculated gravity anomaly profiles allowing for the effect of upwarping with observations along 25¼° E.

occurred before fracture along the centre line the strength of the crust must be of the order of 24×10^8 dyn/cm^2, whereas if it occurred after fracture along the centre line so that the shape is that of the (b) curve of Figure 2 then the strength of the crust is estimated at 13×10^8 dyn/cm^2.

It will be noted from the gravity map that although the anomalies in the region to the west of 22° E are negative, they are very much smaller than those in the east. There are two possible explanations:

(1) that the amount of erosion in the west has been very much less than in the east;

(2) that the cycle is at a much more advanced stage in the west than in the east so that the negative anomaly has been removed by faulting. There is some evidence in favour of the second possibility for it is known that the throw of the fault, marked Worcester fault, on the gravity map is very large. Du Toit (1954) remarks that "The dislocation has near Robertson the enormous throw of probably

not less than 12,000 feet". This estimate is based on work by Söhnge (1935). Recent work by John de Villiers has suggested that the throw is even greater, de Villiers' estimate being 22 500 feet.* It is of interest to note that this is the throw required to remove a root of 6·2 km which was found to be the minimum consistent with the gravity anomalies along the 24½° E and 25¼° E profiles.

7. Other anomalies

The discussion in the preceding sections suggests strongly that in the eroded areas round the coast of South Africa, the cycle of events has been that which could be predicted on the basis of the known properties of the crust and mantle. Unfortunately it is not possible to distinguish between the possibilities that the mantle behaves as a liquid with Newtonian viscosity, or that the effective viscosity is high for low stresses but decreases rapidly once a threshold stress has been reached. In the second case most of the mantle movement would be concentrated in small regions (Hales 1953).

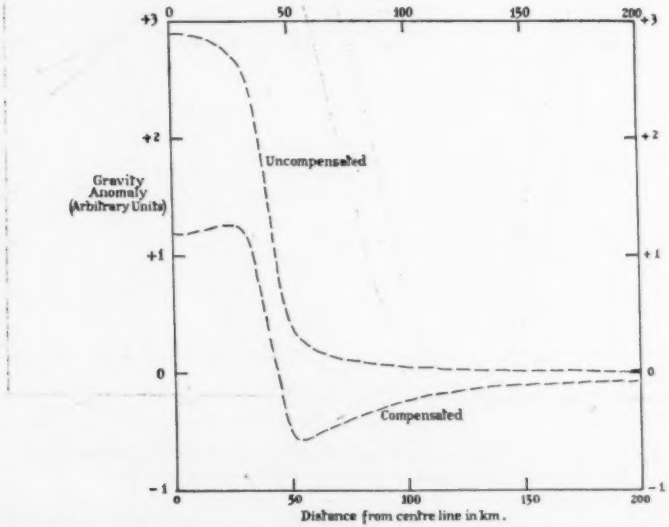


FIG. 7.—Comparison of uncompensated gravity anomaly 80 km wide at a depth of 5 km with the anomaly due to the same structure compensated at a depth of 35 km.

One of the noteworthy features of the gravity map of the Union is a number of positive anomalies. Of these, the anomalies lying in the region bounded by 27° E, 30° E, 24° S and 27° S are known to be associated with the Bushveld Igneous Complex. It had been generally accepted for many years that the Bushveld Igneous Complex was a lopolith and that the granite in the central region was underlain by gabbroic rocks (Molengraaf 1905 and Hall 1932). Recently Truter (1955) suggested that the Complex had originated from several eruptive centres. In a paper dealing with the structure of the mafic portion of the Bushveld Igneous Complex, Cousins (1959) has pointed out that the geological and geophysical evidence, and

* We are indebted to Dr L. E. Kent for this information.

more especially the gravity anomaly pattern, are not consistent with the view that the mafic exposures of the Bushveld Igneous Complex are structurally part of a lopolith. In Cousin's view the mafic rocks of the Complex lie in two deep curved troughs having a superficial synclinal structure. It is clear from the gravity anomalies that the horizontal distribution of the denser mafic rocks must be similar to that suggested by Cousins. With regard to vertical thickness there is some uncertainty for it is not clear whether the dense rocks are compensated or not.

Assuming for the time being that the extra load due to the mafic rocks is uncompensated, one can estimate that the thickness of these rocks is of the order of 7 km for a density difference of 0·3 g/cm^3. However, these rocks are between 1 900 and 2 000 million years old (Schreiner 1958, Nicolaysen, de Villiers, Burger & Strelow 1959). Since a lesser load has resulted in failure of the crust in the Southern Cape, it seems probable that failure will have occurred also in the region of the Complex. In that case the estimate above of 7 km of mafic rocks must be regarded as a minimum estimate. The curves in Figure 7 have been drawn to show the relation between an uncompensated gravity anomaly 80 km wide at a mean depth of 5 km and the anomaly ue to the same structure compensated at a depth of 35 km. From this example it can be inferred that the thickness of the mafic rocks may have been underestimated by a factor of roughly two.

Acknowledgments

We have already acknowledged our indebtedness to the officers of the Geological Survey who made the gravity observations on which the discussion in this paper is based. It remains now for us to express our thanks to Joyce Harris, May Neely and Mavis Scott for help with the computing and in the preparation of the diagrams, and to our colleagues at the Bernard Price Institute for helpful discussions.

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1959 *November* 18.

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The Automatic Reduction of Geophysical Data

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Summary

The automatic recording and reduction of geophysical data is discussed, the results obtained with a proton magnetometer towed behind a ship being used as an example. Data are recorded on punched tape, the sequence of punching being controlled by a uniselector. Each number on the tape is preceded by a symbol that indicates the nature of the quantity recorded; this facilitates the elimination of punching errors and allows quantities to be punched in an arbitrary sequence. The computer programme uses first and second differences to eliminate grossly erroneous readings.

1. Introduction

Substantial savings of time and effort may be obtained by automatically recording observational data in a form that can be accepted by a digital computer. Such a system has been devised for the proton magnetometer used at sea by M. N. Hill (1958) and others. The purpose of this paper is to discuss the methods used to reduce the data from this magnetometer, using them as an example of principles of wider application.

The advantages of an automatic system for recording and reducing observations are not only that the man-hours required are less than if the observations were recorded by an observer and reduced by a human computer, but that there is certainty as to just what has been done to the data and that the observations and results are permanently available in an orderly and systematic form which contains all the relevant information.

It is usually desirable to separate recording and reduction and to reduce to a minimum the equipment added to the measuring instruments. Field equipment is troublesome and expensive to maintain and is more liable to error than a computer. It is therefore best to record raw data in whatever code is most convenient and to leave all logical and arithmetic operations, such as code transformation, zero adjustment and scaling, to the computer.

2. Recording

The output of an instrument is usually a voltage or a rotation of a shaft. This output must be digitized; the method of doing this and the code employed are largely a matter of convenience and of the speed required. Numerous mechanical and electronic devices are available and will not be discussed here. The output from

such a device usually represents the numerical value of the measured quantity by the presence or absence of potentials on a series of bus-bars. The output of the proton magnetometer does not require digitizing since it is in the form of a count held on flip-flops, and is therefore available as potentials on the anodes of the valves. In Hill's magnetometer the output is presented as binary coded decimal digits, each digit being held on four flip-flops representing 1, 2, 4, 8.

The output from a measuring apparatus will usually consist of one or more dependent variables which are to be considered as a function of some independent variable, such as time or distance. A "reading" therefore consists of several numbers all presented simultaneously as a series of binary digits. In the magnetometer we record:

the date and month (4 binary coded decimal digits);
the time in hours and minutes (4 binary coded decimal digits); and
the magnetometer count (6 binary coded decimal digits),

that is, 14 binary coded decimal digits in all.

It would be possible to record the data in a predetermined sequence and allow the computer to determine, by counting, whether the next digit was part of the magnetometer count or part of the date or time. This is a most unsatisfactory method, since the omission of a digit throws out the count and necessitates stopping the computer and correcting or restarting. This difficulty can be avoided by preceding each group of digits representing the magnitude of a physical quantity by a group indicating the nature of the quantity. The use of such indicators gives complete flexibility as to what is recorded, for example readings of some other instrument, such as an echo sounder or a gravity meter, could be recorded along with those of the magnetometer. Also unnecessary repetition of quantities such as the date can be avoided. If readings are taken at uniform intervals of time the reading of the clock need only be recorded occasionally. The field is recorded every minute or half-minute, but the date and time are normally recorded only once an hour. They can however be recorded at any time by depressing a switch; this is necessary at the start of a run or after a break in the series of observations.

The digits which represent a reading are presented simultaneously, but for recording on paper or magnetic tape they must be scanned and fed to the recorder successively. The method employed depends on the speed required. For recording a seismograph one would naturally use electronic gates, but for the magnetometer, which is never read more frequently than twice a minute, a uniselector provides a very simple solution. The anode of one side of each flip-flop was connected to a 1 μF condenser through a 1 Mω resistor. Each decimal digit was therefore represented by the potentials on four condensers. These condensers were connected to a row of contacts on four different arcs of a 25-step 8-bank uniselector. Every time a reading was to be recorded the uniselector was caused to traverse its 25 rows of contacts in about 3 seconds. When an arm of the uniselector encountered a contact connected to a condenser carrying a charge, the condenser was discharged through a relay which caused the corresponding digit to be punched on 5-hole teleprinter tape.* Thus the uniselector causes up to 25 groups of 4 binary digits, each representing a decimal digit or an indicator to be punched in succession on the tape. To avoid 0 being represented by blank tape the fifth hole in each tape row is always punched in any row representing a

* Five holes lying transversely across the tape can be punched simultaneously. We call such a representation of five binary digits a *tape row*.

decimal digit. This enables the digits and the indicators (which never have the fifth row punched) to be distinguished at a glance, and assists the operator to check the proper functioning of the equipment. A magnetometer reading therefore consists of at most 17 5-hole tape rows (14 digits and 3 indicators for date, time and magnetometer count). Of the 25 rows of contacts on the uniselector, 8 are not used and are available for recording other quantities if required.

The use of a uniselector is not always satisfactory. These devices wear out after a few million operations and should therefore only be used where the rate of cycling is so slow that the limited life is acceptable. Also the breaking of currents by uniselector and relay contacts produces sparks and pulses of current that can disturb other equipment. In the magnetometer this is not serious since the recording mechanism only functions after a measurement has been made. In other applications caution would be necessary in mixing such equipment with circuits employing amplifiers.

The impulses to initiate measurement and reading are supplied by a synchronous motor driven by a precision oscillator; this motor also drives a graphical recorder, which gives the operator a plot of the readings as a function of time. Such a record is essential both for the immediate conduct of the measurements and in deciding what shall be done with the data subsequently. The digital clock is operated by impulses supplied once a minute from the synchronous motor. It consists of four 10-step 8-arc uniselectors (digit switches) and provides 4 binary coded decimal digits and a visual indication of its readings on a bank of lights. Circuit diagrams of the whole arrangement can be supplied to those interested, but will not be reproduced here since there are many ways of obtaining the same result.

An analogous system could be used with magnetic tape with one or more tracks. If punched cards were used a large number of digits could be recorded in parallel, and the scanning uniselector would not be necessary. I have no experience with card punches in portable equipment, but would expect trouble from changes in card dimensions due to the wide range of humidity commonly experienced in field work. The bulk of the cards would also be a disadvantage. Paper tape seems a more reliable and convenient medium for the recording of most geophysical data at speeds up to about 10 tape rows per second. At higher speeds a single-track magnetic tape seems the simplest arrangement.

In most series of observations it is necessary to indicate changes in the equipment, such as a resetting of the clock, in order that the computer may take appropriate action in the reductions. For the magnetometer three such indicators are provided, indicating

clock change,
change of interval of measurement, and
end of tape.

These are punched when required by the operator by depressing switches. It was found that the operators made numerous errors in such apparently simple operations as punching these indicators and changing the date at midnight; it is important to arrange such things so that they appear simple and natural to the operator.

The system described has no check on the punching of the digits. This could most easily be provided by using the fifth row of tape as a parity digit (i.e. by punching in the fifth row if the number of holes punched in the other four rows

is even or zero, and not punching if it is odd). Experience shows that punching errors are rare and less frequent than errors due to mal-functioning of the magnetometer. The additional complication of the circuitry does not seem justified to detect a small proportion of the errors, most of which will in any case be removed in the reduction of the data. However, since the occurrence of the "clock change" or "change interval" indicators where they are not intended would upset all later reductions, it is desirable to choose the groups of digits for these indicators so that they cannot be produced by any single punching error in a tape row representing a digit or another indicator.

3. Reduction

An attempt has been made to programme the computer so that it will deal with punching and other errors without the intervention of the operator and will give an indication of the errors found. The ideal is that the machine should take the same action as would be taken by a well-informed person reviewing the data. It has been a matter of some surprise how nearly this ideal can be attained with relatively simple programmes.

The computer performs the following functions:

(a) checks the data for punching errors;

(b) calculates the field, F, from the magnetometer count, C, by the relation

$$F = A/C,$$

where A is a constant;

(c) applies corrections for the magnetic field of the ship (a five-term trigonometric series in the heading), and for the daily variations;

(d) checks that the values of the corrected field and their first and second differences lie between specified limits;

(e) finds the position of the ship, in terms of the distance run along its track, at the time of the measurement;

(f) subtracts a linear function of position to allow for the regional variation of the field; and

(g) smooths slightly and interpolates to equal intervals of distance and prints out the results.

To perform such calculations the machine needs data in addition to those on the tape from the magnetometer. These data are conveniently divided into three parts:

(1) quantities that stay constant for a considerable section of the work, such as the magnetic moment of the proton, the coefficients in the expression for the heading correction, and the normal interval between measurements;

(2) data from the ship's log (data for removing a linear regional trend are also included with this); and

(3) values of the field at a fixed station which can be used to obtain the correction for the daily variation. Hourly values are used and interpolated using second differences.

In deciding the form in which to provide such data the convenience and habits of the provider of the data should be given priority over those of the machine or its programmer. For example, it is usual in a ship's log to use "ship's time" which will often differ from the time kept by the clock in the magnetometer or at the fixed magnetic station. The programme should allow the data from the log to be

tabulated in terms of ship's time and should provide for the ship's clocks being put forward or back at any time, without however requiring the difference of ship's and Greenwich time to be stated with every entry.

The first step in reducing the data is for the computer to read and interpret an indicator and to verify that it is followed by the proper number of valid binary coded decimal digits and by another indicator. If any digit is missing or defective or if there is an extra digit, the reading is rejected, but if the rejected quantity is a magnetometer reading it is included in the count that the machine keeps to determine the time. The cause of rejection is indicated by a digit punched by an output punch reserved for that purpose (EDSAC, the machine used for the work, has two punches). This punch also records the reading of a "clock change" indicator so that it is possible to verify that no spurious ones are present.

The computations required are straightforward and the main interest of the programming is in the removal of erroneous observations. The errors are largely due to electrical noise causing the gates in the counters of the magnetometer to function at the wrong time. When everything is working well they constitute less than 1 per cent of the observations, but sometimes, for reasons that are obscure, they rise to as much as 10 per cent. A few will be due to punching errors that have not been detected in the initial checking. Whilst any automatic system will inevitably produce some grossly erroneous values which must be eliminated in the reductions, care is necessary not to set such strict criteria as to reject observations indicating real charges in the measured quantity or forming part of the normal distribution of errors of measurement.

Some of the errors can be removed by setting upper and lower limits outside which the field cannot go. This will, for example, eliminate the zero readings obtained if the punch is left on while the magnetometer is turned off. Less gross errors are best eliminated by considering the smoothness of the run of successive readings. This is the criterion that a human reviewer of the data would apply. The eye judges an observation plotted on a graph in relation to a number of preceding and following ones. A process using any number of observations could be programmed, but experience has shown that for the magnetometer records it is sufficient to consider an observation in relation to its neighbours on each side. If an observation F_n taken at time t_n is being tested, the observation F_{n-1} at t_{n-1} has already been tested and is regarded as acceptable, but either F_n or the succeeding observation, F_{n+1} taken at t_{n+1}, may be erroneous.

If both the differences $|F_n - F_{n-1}|$ and $|F_{n+1} - F_n|$ are less than a specified constant l (usually taken as 25γ when observations are taken every half-minute), and the intervals between the three observations are the normal ones, the observation F_n is accepted if the second difference, $|F_{n+1} - 2F_n + F_{n-1}|$, is also less than l. If one or both the intervals are not the normal ones, as may happen if observations have been rejected between t_{n+1} and t_{n-1}, the second difference check is omitted unless $|F_{n+1} - F_{n-1}|$ is less than l. Thus the second difference check can be applied over a short gap when the field is varying slowly, this will occasionally result in a good observation being rejected, but considerably strengthens the check on observations whose neighbours have been rejected.

If the first difference before an observation, $|F_n - F_{n-1}|$, is less than l, and that after it, $|F_{n+1} - F_n|$, is greater than l, the observation is accepted without a second difference check. This provision is necessary to prevent a faulty F_{n+1} causing the rejection of F_n. As F_{n-1} has already been accepted, an observation for which $|F_{n+1} - F_n|$ is less than l, but $|F_n - F_{n-1}|$ is not, is rejected unless there is a missing

observation between F_{n-1} and F_n; this last provision is necessary to prevent the rejection of an observation following a gap.

These rules and the choice of l have been developed gradually by comparing the rejections with those decided on by plotting and examining long series of data. Such a scheme is, of course, only effective if there is a considerable redundancy in the data; an observation can only be judged defective if the rest of the observations give a good idea of what its value should be. The scheme described requires an observation to be checked by its two neighbours and therefore requires that at least 50 per cent more information can be collected than is strictly necessary. The usual spacing of observations used for plotting profiles in geophysics is believed to provide at least this degree of redundancy.

The limit for the first and second differences has been taken as a constant; it would be possible to make it depend on the smoothness of the preceding observations and to use a lower limit when the field is varying smoothly. This is not desirable with the magnetic observations as a long run of small differences may suddenly come to an end as the ship runs over some geological discontinuity, and the small limit inherited from the preceding part of the track would cause unjustified rejection of the observations indicating the kind of feature the survey is designed to find.

The causes of rejection are indicated by digits punched along with those indicating the rejections for faulty punching. To enable an error on the magnetometer tape to be located each group of these digits is succeeded by the time and ship's position for the next satisfactory observation.

An alternative method, which has been tried and found workable in another problem, is to fit a suitable function to a number of observations before and after the one to be tested and to reject if the departure from the fitted function exceeds a given amount, or a given number of times the standard deviation of the other observations. Such a method is not satisfactory if some gross errors are present, since these will upset the fitted function and may lead to the wholesale rejection of satisfactory observations.

Methods of detecting errors that depend on the smoothness of the run of the observations are usually satisfactory for quantities that constitute a time series or represent a continuous profile in space. They cannot however be used where the function observed is discrete and there is no correlation between successive readings. This is true, for example, in measuring the darkening in the photographic images of stars, or in the readings of a series of peaks in an X-ray diffraction pattern. It should usually be possible to use regularities among the data (such as crystal symmetry) or to repeat the measurements in slightly different circumstances (e.g. after inverting the plate carrying the star images). In such measurements the redundancy may not be great and it may be necessary to substitute good observations for the defective ones. It would then be desirable to check by a programme separate from the one that produces the final results.

It is sometimes possible to correct errors. For example, at one time the magnetometer occasionally missed exactly one count out of 2048. This resulted in a second difference very close to $1/1024$ of the field. The programme was therefore made to test whether a second difference greater than l was within 3γ of $F_n/1024$ and if it was to add $F_n/2048$ to F_n. Little is gained by such correction since almost as good a value could be obtained by interpolation among the acceptable measurements.

The corrected values of the magnetic field, with the regional trend removed,

are required either as a function of time or of distance along the ship's track. The former is usually more convenient if the results are to be plotted and contoured on a chart where the track is already marked off in time. For the preparation of sections, however, the independent variable must be distance. The results are therefore interpolated by fitting a quadratic by least squares to four successive values, this provides a slight smoothing and bridges short gaps due to rejected observations. The punching of the output takes much longer than the reading of the input tapes or the computation; it is therefore important to eliminate unnecessary punching of spaces and non-significant digits.

The sections of the track to be dealt with and the interval of interpolation are specified on a separate tape.

It is important that the printed sheets made from the output tapes should give all the relevant information. An output tape from the magnetometer reduction opens with the name of the version of the programme used, the name of the ship and of the fixed station, and the month and year. It then gives a pair of letters indicating a section of track and the results. These are followed by the date, the number of acceptable measurements, the clock error and the interval of measurement, the mean and variance of the results, and the values of the field at the ends of the section of track used in removing the regional trend. It is important that the output should contain not only the results but also sufficient information to provide an easy check that the correct data has been supplied to the machine. The second tape listing the causes of rejection of observations is rarely looked at, its punching can be suppressed to save machine time.

4. Further treatment of data

The corrected and interpolated output can be used as data for further computations. Examples of processes that might be required are: smoothing; fitting of a least squares polynomial; auto- or cross-correlation; Fourier transformation; estimation of the power spectrum. Programmes for most of these are available, but experience shows that much time is wasted in adapting and combining them to fit particular data and requirements. The ideal would be to regard the programmes for reducing the data and the programmes for analysing and transforming it as a system of sub-routines controlled by a simple master programme specifying, in some suitable language, what is to be done to what data. It is hoped to design such a system.

5. Acknowledgments

The magnetic results for which this system was devised were obtained by Dr N. M. Hill and his collaborators, particularly by Mr T. D. Allan. I have greatly benefited from their advice.

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The Condition that a Long-Period Tide shall follow the Equilibrium-Law

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Summary

It is only when friction is taken into account that the limiting form of a long-period tidal constituent always follows the equilibrium-law. Hence the question arises as to how long the period of a constituent must be in order that its distribution may be given approximately by the equilibrium-law. In this paper reasons are given for believing that:

(i) the constituent whose period is nearly 19 years will certainly follow the equilibrium-law;
(ii) the semiannual and annual constituents will probably follow the equilibrium-law;
(iii) the fortnightly and monthly constituents will probably not follow the equilibrium law.

1. Introduction

Because, in the ocean, it is possible for currents to exist with a stationary ocean-surface, the limiting forms of frictionless tides of long period are not always given by the equilibrium-law.

In a paper "On the dynamical theory of the tides of long period", G. H. Darwin (1886) said, "In treating these oscillations Laplace . . . seeks to show that friction suffices to make the ocean assume at each instant its form of equilibrium. His conclusion is no doubt true, but the question remains as to what amount of friction is to be regarded as sufficing to produce the result, and whether oceanic tidal friction can be great enough to have the effect which he supposed it to have. . . . The quickest of the tides of long period is the fortnightly tide, hence for the applicability of Laplace's conclusion the modulus of decay* must be short compared with a week. Now it seems practically certain that the friction of the ocean bed would not much affect the velocity of a slow ocean current in a day or two. Hence we cannot accept Laplace's hypotheses as to the effect of friction. . . . There is one tide, however, of long period of which Laplace's argument from friction must be true. In consequence of the regression of the nodes of the Moon's orbit there is a minute tide with a period of nearly nineteen years and in this case friction must be far more important than inertia."

In his *Théorie des marées* (1910) Poincaré termed tides which follow the equilibrium-law "marées statiques de la première sorte" and the other limiting forms "marées statiques de la deuxième sorte". Then, following S. S. Hough (1896),

* $2/k$ in the notation of Section 2 below

he said: "Nous verrons qu'il faut une dizaine d'années pour que le frottement puisse se faire sentir; par consequent, les marées annuelles et de periode plus courte seront bien de la deuxième sorte; au contraire, la marée ayant pour periode 18 ans serait une marée de première sorte, qu'il devrait calculer par la théorie de l'equilibre".

The object of this Note is to show that the conclusion of Darwin and Poincaré regarding the tidal constituent with a period of nearly 19 years is correct, but that Poincaré's statement regarding the annual constituent is probably not correct.

Hough and Poincaré supposed that tidal friction arose through molecular viscosity and followed a linear law. In the present Note it is supposed that total friction arises through eddy-viscosity near the ocean-bed and follows a quadratic law. But by supposing that the long-period constituent is superposed on a much larger short-period constituent, we arrive at a linear law for the long-period constituent with a coefficient which is proportional to the maximum value of the short-period current (Bowden 1953).

2. Law of bottom-friction

Let the bottom-currents of the long- and short-period constituents be denoted respectively by

$$v, \quad V\cos\sigma_0 t,$$

where t denotes the time, V, σ_0 are constants and v is treated as a constant over a number of short periods. Then the force of bottom-friction is taken as

$$0{\cdot}0025\rho|V\cos\sigma_0 t+v|(V\cos\sigma_0 t+v), \tag{1}$$

ρ denoting the density, and Bowden's paper shows that the frictional term in the ordinary equation of motion for the long-period constituent will be

$$kv, \text{ where } k=\frac{1}{100\pi}\frac{V}{h}. \tag{2}$$

One would expect that when the period of this constituent is long compared with the frictional period $2\pi/k$ the constituent will follow the equilibrium-law, and that when the period is short compared with $2\pi/k$ the constituent will follow the limiting form with currents. This criterion is supported by the analysis of the following section.

3. Dynamical equations

In 1916 I transformed the dynamical equations of the tides, for any sea or ocean, into a form using an infinite number of discrete coordinates. I then neglected friction, but, if k of (2) is treated as uniform, friction may easily be taken into account.

For a tidal system with *two degrees of freedom*, the equations are

$$\ddot{p}+k\dot{p}+\beta\dot{q}+\nu^2 p = P \tag{3}$$

$$\ddot{q}+k\dot{q}-\beta\dot{p} = 0, \tag{4}$$

where p, q, P are functions of time and k, β, ν^2 are constants. Here p determines the elevation of the sea-surface, q determines that part of the current which is

possible with a stationary elevation, P represents the tide-generating forces, k is the coefficient of friction of (2), β a geostrophic coefficient due to the rotation of the earth, and ν^2 a coefficient of stability depending on the Earth's gravity.

The *equilibrium-law* is given by

$$\dot{p} = \dot{q} = 0 \qquad \text{so that} \qquad \nu^2 p = P. \tag{5}$$

On neglecting friction, the equation (4) gives

$$\ddot{q} = \beta \dot{p}$$

and integration of this equation gives

$$\dot{q} = \beta p + \text{constant}. \tag{6}$$

When the motion is periodic, the constant of (6) vanishes. The *limiting form of long-period tides* is then given by (3) with

$$\dot{p} = 0, \quad \dot{q} = \beta p$$

so that

$$(\nu^2 + \beta^2) p = P. \tag{7}$$

For complex harmonic motion with time-factor $e^{i\sigma t}$, the equations (3), (4) become

$$(-\sigma^2 + i\sigma k + \nu^2) p + \beta \dot{q} = P \tag{8}$$

$$(i\sigma + k) \dot{q} = i\sigma \beta p \tag{9}$$

so that, on eliminating $\dot{q}$,

$$\left\{ -\sigma^2 + i\sigma k + \nu^2 + \frac{i\sigma}{i\sigma + k} \beta^2 \right\} p = P. \tag{10}$$

This indicates that the form of the long-period tides depends on the value of

$$\frac{i\sigma}{i\sigma + k} \tag{11}$$

when σ is small. When

$$k \neq 0, \quad \sigma = 0$$

the equation (10) gives the equilibrium-form (5); when

$$\sigma \to 0, \quad k/\sigma \to 0$$

the equation (10) gives the limiting form with currents (7).

For *four degrees of freedom* the equations, with an obvious extension of the notation of (3), (4), are

$$\ddot{p}_1 + k\dot{p}_1 + \beta \dot{p}_2 + \beta_{1,1}\dot{q}_1 + \beta_{1,2}\dot{q}_2 + \nu_1^2 p_1 = P_1 \tag{12}$$

$$\ddot{p}_2 + k\dot{p}_2 - \beta \dot{p}_1 + \beta_{2,1}\dot{q}_1 + \beta_{2,2}\dot{q}_2 + \nu_2^2 p_2 = P_2 \tag{13}$$

$$\ddot{q}_1 + k\dot{q}_1 + \beta' \dot{q}_2 - \beta_{1,1}\dot{p}_1 - \beta_{2,1}\dot{p}_2 = 0 \tag{14}$$

$$\ddot{q}_2 + k\dot{q}_2 - \beta' \dot{q}_1 - \beta_{1,2}\dot{p}_1 - \beta_{2,2}\dot{p}_2 = 0. \tag{15}$$

For stationary motion

$$\dot{p}_1 = \dot{p}_2 = 0, \quad \ddot{q}_1 = \ddot{q}_2 = 0$$

so that the equations (12)–(15) become

$$\beta_{1,1}\dot{q}_1+\beta_{1,2}\dot{q}_2+\nu_1^2 p_1 = P_1 \tag{16}$$

$$\beta_{2,1}\dot{q}_1+\beta_{2,2}\dot{q}_2+\nu_2^2 p_2 = P_2 \tag{17}$$

$$k\dot{q}_1+\beta'\dot{q}_2 = 0 \tag{18}$$

$$k\dot{q}_2-\beta'\dot{q}_1 = 0. \tag{19}$$

From (18), (19) it follows that, unless

$$k^2+\beta'^2 = 0,$$

then

$$\dot{q}_1 = \dot{q}_2 = 0,$$

and then (16), (17) give

$$\nu_1^2 p_1 = P_1, \quad \nu_2^2 p_2 = P_2 \tag{20}$$

which is the equilibrium-form. The condition that the limiting form of frictionless tides shall not be the equilibrium-form is thus

$$\beta' = 0. \tag{21}$$

When the condition (21) is satisfied and the motion is complex harmonic, the equations (12)–(15) give

$$(-\sigma^2+i\sigma k+\nu_1^2)p_1+i\sigma\beta p_2+\beta_{1,1}\dot{q}_1+\beta_{1,2}\dot{q}_2 = P_1$$

$$(-\sigma^2+i\sigma k+\nu_2^2)p_2-i\sigma\beta p_1+\beta_{2,1}\dot{q}_1+\beta_{2,2}\dot{q}_2 = P_2$$

$$(i\sigma+k)\dot{q}_1 = i\sigma(\beta_{1,1}p_1+\beta_{2,1}p_2)$$

$$(i\sigma+k)\dot{q}_2 = i\sigma(\beta_{1,2}p_1+\beta_{2,2}p_2)$$

so that the elimination of $\dot{q}_1$, $\dot{q}_2$ gives

$$\left\{-\sigma^2+i\sigma k+\nu_1^2+\frac{i\sigma}{i\sigma+k}(\beta_{1,1}^2+\beta_{1,2}^2)\right\}p_1 + i\sigma\beta p_2+\frac{i\sigma}{i\sigma+k}(\beta_{1,1}\beta_{2,1}+\beta_{1,2}\beta_{2,2})p_2 = P_1 \tag{22}$$

$$\left\{-\sigma^2+i\sigma k+\nu_2^2+\frac{i\sigma}{i\sigma+k}(\beta_{2,1}^2+\beta_{2,2}^2)\right\}p_2 - i\sigma\beta p_1+\frac{i\sigma}{i\sigma+k}(\beta_{2,1}\beta_{1,1}+\beta_{2,2}\beta_{1,2})p_1 = P_2. \tag{23}$$

Again, the single factor

$$\frac{i\sigma}{i\sigma+k}$$

is a criterion as to how far the solution approximates to the equilibrium-form. When

$$k \neq 0, \quad \sigma = 0$$

(22), (23) become respectively

$$\nu_1^2 p_1 = P_1, \quad \nu_2^2 p_2 = P_2 \tag{24}$$

which give the equilibrium-form. When

$$\sigma \to 0, \quad k/\sigma \to 0$$

(22), (23) become respectively

$$(\nu_1^2+\beta_{1,1}^2+\beta_{1,2}^2)p_1+(\beta_{1,1}\beta_{2,1}+\beta_{12}\beta_{2,2})p_2 = P_1 \tag{25}$$

$$(\nu_2^2+\beta_{2,1}^2+\beta_{2,2}^2)p_2+(\beta_{2,1}\beta_{1,1}+\beta_{22}\beta_{1,2})p_1 = P_2 \tag{26}$$

which determine the limiting form with currents.

For *any number of degrees of freedom* it is clear that a sufficient condition for the limiting form not to be the equilibrium-form is that all the geostrophic coefficients between pairs of q's are zero. When this condition is satisfied, it is also clear that the factor

$$\frac{i\sigma}{i\sigma+k}$$

will enter the solution exactly as in the cases of two and four degrees of freedom.

4. Numerical estimates

From (2)

$$\frac{2\pi}{k} = 200\pi^2\frac{h}{V} = 2\times 10^3\frac{h}{V} \tag{27}$$

very approximately. But h/V varies enormously over the ocean, while the formulae of Section 4 imply a uniform value of k. It is therefore only possible to make a rough estimate of the effective value of $2\pi/k$.

In shallow water, we might have

$$h = 100\,\text{m}, \quad V = 50\,\text{cm/s}$$

so that

$$\frac{h}{V} = \frac{100\,\text{m}}{50\,\text{cm/s}} = 200\,\text{s}$$

and

$$\frac{2\pi}{k} = 4\times 10^5\,\text{s} = 4{\cdot}6\,\text{d}.$$

In mid-ocean, we might have

$$h = 4000\,\text{m}, \quad V = 5\,\text{cm/s}$$

so that

$$\frac{h}{V} = \frac{4000\,\text{m}}{5\,\text{cm/s}} = 8\times 10^4\,\text{s}$$

and

$$\frac{2\pi}{k} = 16\times 10^7\,\text{s} = 5{\cdot}1\,\text{years}.$$

The effective value of the frictional period $2\pi/k$ will be between 4·6 d and 5·1 years.

If we suppose that shallow water extends over one-tenth of the ocean, and take k one-tenth of the value for shallow water, we have

$$\frac{2\pi}{k} = 46\,\text{d}.$$

If this estimate is valid, it will follow that the fortnightly and monthly constituents will not follow the equilibrium-law, but that the semiannual and constituents of longer periods will follow the equilibrium-law. The conclusion about the 19 yearly constituent appears to be quite safe, as its period is long, even compared with the larger frictional period of 5·1 years.

Edgemoor,
Verwood,
Dorset:
1959 *December.*

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PKIKP and Pseudo-*PKIKP* Phases at distances of less than 140°*

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Summary

In several attempts to establish from observations the travel-time curve for *PKIKP* (called *P'* or *P''* by some authors) the beginning of this curve has been drawn with a relatively strong curvature corresponding to a decrease of $dt/d\theta$ from roughly 4 s/deg at $\theta = 110°$ to about 2 s/deg at 130°. Apparently, travel times of different wave types had been combined. Actually, the observed travel-time curve of *PKIKP* is nearly a straight line in agreement with calculations. Near an epicentral distance of 115° the amplitudes of *PKIKP* increase by a factor of roughly 5. This probably indicates the transition from the diffracted to the direct *PKIKP*.

The portions of the travel-time curves with a slope corresponding to $dt/d\theta$ near 4 s/deg which erroneously had been attributed to *PKIKP* belong to several wave types which seem to follow *P*, *pP* and *sP* or precede *PP*, each at roughly constant time intervals. Among causes for such multiplicity of phases are effects of irregularities in the Earth, of discontinuities and of diffraction.

1. The problems

Extensive travel times of *PKIKP* at distances of between about 110° and 140° seem to have been collected first by Rudolph and Szirtes (1914) who stated in a preliminary note based on observations of 74 earthquakes 1906–1913 that at an epicentral distance of about 112° a travel-time curve branches from that of *PP*, and that this curve can be followed to the anticentre where these waves arrive about 1½ min later than at 112°. However, the reported travel times of 11 min at 112° and 12½ min at 180° seem to contain a systematic error of about 8 min; most reported times and interpretations of phases by Rudolph and Szirtes are incorrect. Apparently, the war and Rudolph's death prevented a detailed publication of their results.

The first detailed observed travel-time curve of *PKIKP* at distances of less than 140° has been published by Angenheister (1921); he designated the longitudinal waves through the core by *P'*. Later, the symbol *P''* has been used occasionally for *PKIKP* at distances of less than 143°. This procedure is followed

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† Died 1960 January 25.

here in some of the illustrations. Most travel-time curves and tables for *PKIKP* at distances of less than 140° which had been published between 1922 and 1933 are based mainly on Angenheister's data (condensed in Table 1). In addition to various investigations based on specific shocks, in which some data for the times of *PKIKP* have been given, specific investigations of the travel times of *PKIKP* have been published by Gutenberg & Richter (1934, p. 90) and by Jeffreys (1938, 1939a, 1939b).

Table 1

Travel times t of PKIKP in min:s according to various authors:
h = focal depth in km, θ = distance in degrees

Author		*h*	θ = 105	110	115	120	125	130	135
			(a) Observed						
Angenheister (1921)		30	18:05	18:29	18:50	19:07	19:22	19:34	19:45
Gutenberg–Richter (1934)	No. 2	30	—	18:00	18:20	18:37	18:53	19:04	19:12
Gutenberg–Richter (1934)	No. 3	30	18:00	18:15	18:31	18:47	18:59	19:10	19:17
Denson (1950)		30	—	18:15	18:39	18:54	19:05	19:14	19:22
Bolt (1959)*		0	—	18:35	18:44	18:54	19:04	19:13	19:23
Gutenberg (1959)		600	17:17	17:27	17:38	17:49	17:59	18:09	18:18
			(b) Calculated from assumed velocity as function of depth						
Jeffreys–Bullen (1940)		0	—	18:33	18:43	18:53	19:02	19:12	19:21
Gutenberg (1958)†		0	—	(18:36)	18:46	18:56	19:06	19:15	19:24
Jeffreys–Bullen (1940)‡		600	—	17:26	17:36	17:46	17:55	18:05	18:14
Gutenberg (1958)†‡		600	—	(17:29)	17:39	17:49	17:59	18:08	18:17

* $t = 19{:}03{\cdot}7 + 1{\cdot}92\,(\theta - 125°)$s.
† Based on Table 4 in Gutenberg (1958, p. 309) for times in the core.
‡ Assumed correction for depth −1:07.

Originally it had been believed that *PKIKP* waves arriving at distances of between 110° and about 142° are the result of longitudinal waves diffracted through the core. The main progress resulted from the suggestion by Lehmann (1936) that these waves are a consequence of a sudden or rapid increase in the velocity inside the core. Subsequently, Gutenberg & Richter (1938) have given a specific solution with a gradual transition from the outer to the inner core which later has been improved by Gutenberg (1958a; 1959, pp. 105–109). Jeffreys (1939b), on the other hand, has supposed that the transition is sudden. However, to fit the data, he had to assume that approaching the inner core there is a decrease in velocity which is not indicated otherwise (see, e.g. Gutenberg, 1959, pp. 110–113).

Jeffreys (1939a) has pointed out that on the basis of Airy's theory the waves diffracted at the caustic of *PKP* could not be observable at a distance of much beyond 10° preceding the caustic. This has been confirmed by observations (Gutenberg 1958b). In addition to the problem of the type of transition from the outer to the inner core which is not discussed here, there are the questions, at what minimum epicentral distance do *PKIKP* waves arrive through the inner core, and what form the observed travel-time curve has of diffracted *PKIKP* waves, arriving at shorter distances.

2. The beginning of *PKIKP*

Travel-time curves based on observed *PKIKP* waves and constructed without reference to theoretical curves begin at epicentral distances slightly over 100° where small waves are found preceding *PP*. As soon as theoretical curves of *PKIKP* were based on the conception of an inner core it appeared that *PKIKP* waves could not arrive at distances of less than somewhere between about 110° and 120°. The complicated travel-time curves including a loop with several branches differing by small time intervals (see, e.g. Jeffreys 1939a, p. 358) make accurate determinations very difficult. This can be seen also in Figure 1, which

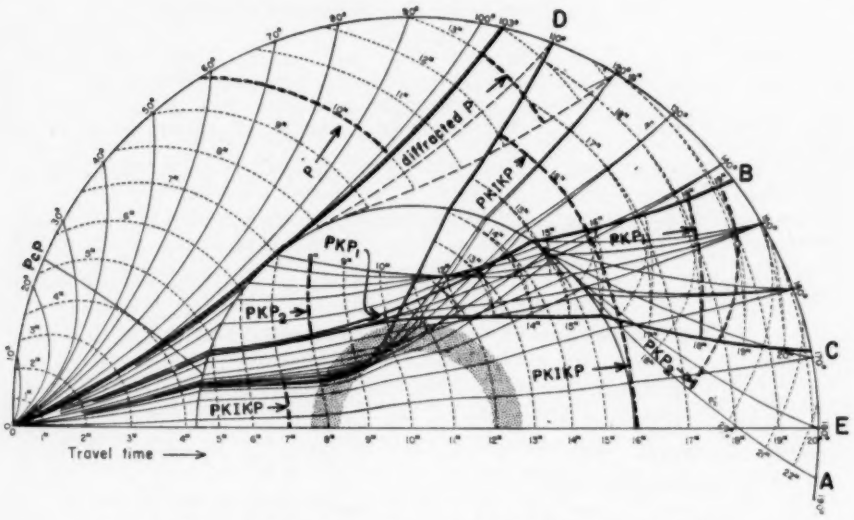

FIG. 1.—Wave paths and wave fronts (travel times) of longitudinal waves in the Earth. Based on Gutenberg & Richter (1939, p. 123).

shows in addition that *PKIKP* waves arriving at distances of less than roughly 140° have nearly the same angle of incidence, corresponding to their nearly straight line travel-time curve. The amplitudes of these waves should be rather small since relatively little energy leaves the source between the rays arriving at the rather wide range of distances between about 110° and 140° (*D* and *B* in Figure 1). Nevertheless, it may be expected that a noticeable increase in amplitudes is observable at the distance corresponding to *D* in Figure 1.

Denson (1952, p. 130) studied the amplitudes of *PKIKP* as function of the epicentral distances. From his results, it appears that a/T (a = ground amplitude in microns, T = period in s) decreases noticeably at distances θ between 110° and 120°. In shocks of magnitude 7·0, a/T begins to exceed 1 μm/s in the vertical component of short-period *PKIKP* at about $\theta = 116°$. The corresponding long-period waves show slightly smaller values of a/T. Since records of the earthquake of 1957 April 16, at 4:04:04, adopted source at 4½° S 107½° E at a depth of 600 km, magnitude $m = 7{\cdot}2$ have been very useful in the investigation of several problems, values of a/T have been calculated from the records of vertical components of *PKIKP* at stations for which instrumental constants have been

available. The results show that at distances of less than 110° (Relizane, 106°·4; Rathfarnham, 107°·7; Toledo, 109°·3) a/T is $\frac{1}{4}$ to $\frac{1}{2}$ μm/s, while at distances of between 110° and 129° (Malaga, 110°·3; Alberni, 117°·3; Horseshoe Bay, 118°·1; Victoria, 118°·5; Banff, 121°·3; Santa Clara, 124°·3; Pasadena, 128°·4) its value is between $1\frac{1}{4}$ and 3 μm/s. However, we must consider that in a 600 km deep shock the distances at which *PKIKP* waves arrive with a given angle of incidence are one to two degrees shorter than in shallow shocks. Combining the new results with those of Denson it seems that in shallow shocks of $m = 7{\cdot}0$ *PKIKP* begins to have values of a/T near 1 μm/s at a distance of roughly 115° and that at this distance possibly the actual *PKIKP* waves begin. However, there is no well-defined caustic anywhere near this distance; compare Figure 2. The

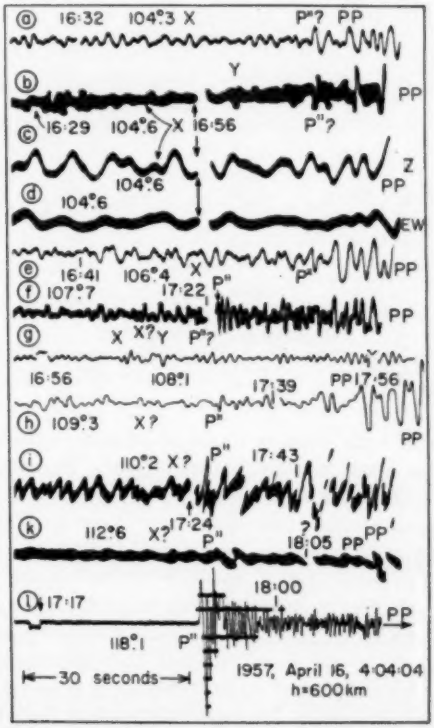

FIG. 2.—Portions of seismograms showing *PKIKP* (= *P″*) at epicentral distances between 104° and 119° on records of shock of 1957 April 16, 4: 04: 04, focal depth 600 km, all on the same time scale. (d) is an East–West component; the others are vertical components. The recording stations are (a) Alger, (b) to (d) Kew, (e) Relizane, (f) Rathfarnham, (g) Sitka, (h) Toledo, (i) Malaga, (k) Coimbra, (l) Horseshoe Bay

reason is probably the relatively small energy going into these *PKIKP* waves near the source (Figure 1).

3. The diffracted *PKIKP* waves

All early travel-time curves for the observed *PKIKP* waves start at a distance of about 105° with a slope nearly parallel to that of *P* or of *PP*. At distances

between about 115° and 125° the apparent velocity corresponding to these curves gradually increases and at distances beyond 130° this slope corresponds to about that to be expected from calculations on the basis of velocity–depth curves. Tables 1 and 2 illustrate these facts. The travel-time curve for a focal depth of 600 km, given by Gutenberg & Richter (1936, p. 350), had been based on the observed travel-time curve for shallow shocks (Gutenberg & Richter 1934) and therefore gives a decrease in $dt/d\theta$ from about 3·4 s/deg. at $\theta = 107°{\cdot}5$ to 1·8 at 127°·5. Very recent travel-time curves do not contain observations of waves giving $dt/d\theta$ noticeably in excess of 2 s/deg.

Table 2

Values of $dt/d\theta$ in s/deg., based on Table 1.
Symbols as in Table 1

Author	h	$\theta = 107\frac{1}{2}$	$112\frac{1}{2}$	$117\frac{1}{2}$	$122\frac{1}{2}$	$127\frac{1}{2}$	$132\frac{1}{2}$
(a) From observed curves							
Angenheister (1921)	30	4·8	4·2	3·4	3·0	2·4	2·2
Gutenberg–Richter (1934) No. 2	30	—	4·0	3·4	3·2	2·2	1·6
Gutenberg–Richter (1934) No. 3	30	3·0	3·2	3·2	2·4	2·2	1·4
Denson (1950)	30	—	4·8	3·0	2·2	1·8	1·6
Bolt (1959)	0	1·92	1·92	1·92	1·92	1·92	1·92
Gutenberg (1959)	600	2·0	2·2	2·2	2·0	2·0	1·8
(b) from calculated curves							
Jeffreys–Bullen (1940)	0	—	2·0	2·0	1·8	2·0	1·8
Gutenberg (1958)	0	—	(2·0)	2·0	2·0	1·8	1·8

The change in $dt/d\theta$ with distance in the older observed travel-time curves for *PKIKP* (Table 2) has puzzled many seismologists. This change cannot be explained theoretically. In several instances (e.g. Jeffreys 1938, p. 299) observations of *PKIKP* at distances of less than about 120° have been disregarded. "Even between 120° and 142°·5 the readings are scattered and may not at all refer to the same phenomenon" (Jeffreys 1938, p. 299).

Records of the shock of 1957 April 16, 4: 04: 04 offer an opportunity to investigate which observed travel times of phases near *PKIKP* belong to one and the same curve on the basis of the form, periods and amplitudes of the respective waves. Figure 2 shows examples of records, Figure 3 gives travel times which are based on readings by the author. Apparently, several wave types, two of them marked by *X* and *Y* respectively, follow the *P* group (*P*, *pP*, *sP*). Others precede *PP*, as it has been discussed by Gutenberg (1960). At distances where *P*, *pP* and *sP* become small, the related later waves also decrease and finally disappear. On the other hand, at the distances where *PKIKP* and *PKP* become large, they are followed by more and more "unexplained" phases (cf. Figure 3). Figure 4 gives examples of such seismograms. The additional phases are better developed on records of short-period instruments than on those of long-period. Figure 4a shows an example of similar unexplained phases following *P*. In many instances such waves are so large and clear that they may lead to misinterpretation of records. Frequently their travel-time curves seem to follow well-defined travel-time curves (cf. Figure 3), but good judgment is needed to discriminate between probable and accidental alignment along reasonable curves. Apparently, in several

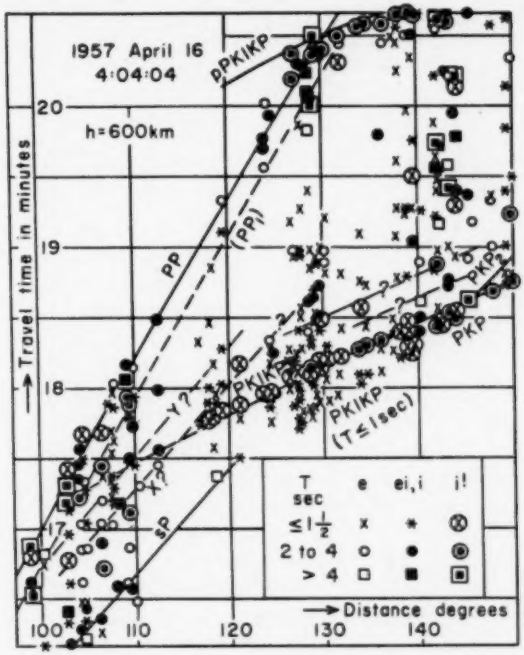

FIG. 3.—Observed travel times and travel-time curves for shock of 1957 April 16, 4: 04: 04, focal depth 600 km.

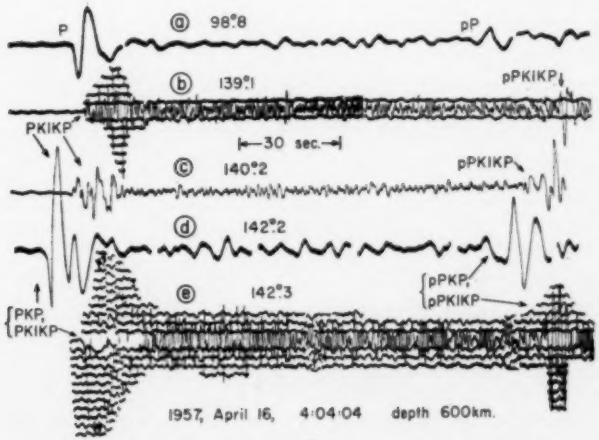

FIG. 4.—Portions of vertical components recorded from the same shock as those in Figure 2. The records, showing multiplicity and unexplained phases, are on the same time scale; (a) shows portion between *P* and *pP* recorded at Stuttgart; (b) to (e) portions between *PKIKP* and *pPKIKP*. The recording stations are (b) Montreal, (c) Lubbock, (d) Florissant, (e) Weston; in (e) the maxima of *PKIKP* and *PKP*, which coincide, are cut off on both sides.

of the travel-time curves for observed *PKIKP* waves (Table 1), the beginning of the curve for *X* (Figures 2 and 3) had been connected with the curve for *PKIKP* at somewhat greater distances. The new observed curve for *PKIKP* corresponds closely to the calculated curve for a focal depth *h* of 600 km.

4. Reasons for unexplained travel-time curves and for motion between established phases

The problem of "multiplicity" of phases on records has been studied already by A. Mohorovičić (1914). Later, Gutenberg & Richter (1934, p. 129) expressed "considerable doubt as to whether the hypothesis of dispersion is adequate to account for the observations". Benioff (oral communication) has found examples of records of near-by shocks from the same source, in which practically every detail of each wave is duplicated. Consequently, for shocks with a simple displacement at the source the whole complicated motion recorded at a given distant station must depend on processes along the path of the waves.

Irregularities in the structure of the Earth which are of the magnitude of the wave lengths involved play a role in the creation of scattered waves following the main phases. Tatel (1954) has pointed out that centres of scattering at a surface may cause conversion from Rayleigh waves to longitudinal waves. "The complex nature of a seismogram can be attributed in part to the interaction of seismic waves with surface irregularities." Moreover, there are various types of diffracted waves which are not found on the basis of the usual wave theory in which terms of second and higher order are neglected. Waves reflected at discontinuities in and near the crust, especially at the Mohorovičić discontinuity, are an additional source of "unexplained" phases (Gutenberg 1960). Another source of multiplicity which has been mentioned already by A. Mohorovičić (1914, p. 157) is provided by the fact that at each discontinuity, especially those in and near the crust, reflected and refracted transverse waves originate, if a longitudinal wave is incident, and vice versa.

As A. Mohorovičić (1914, p. 157) has already pointed out, the establishing of travel-time curves requires the use of original records or good copies so that the individual phases can be studied and compared. Reported arrival times may lead to incorrect conclusions concerning the travel-time curve of a given phase as in the case of the "observed" travel-time curves of *PKIKP*.

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California Institute of Technology,
Pasadena, California:
1959 *December.*

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A New Method of Deriving Seismic Velocity Distributions from Travel-time Data

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Summary

A law of the form $v = ar^{\zeta_0}$ gives a useful first approximation to the seismic velocity distribution in various parts of the Earth. A method is here devised of finding the corrections to numerical values given by the law which are needed to fit a given set of seismic travel-time data. The method provides an alternative to the classical Herglotz method of determining seismic velocity distributions, and can be specially powerful in regions where the proportionate deviations from the above velocity law are small or moderate. The method is exact in spite of the non-linear form of the first approximation. Auxiliary notes are given which facilitate computation in particular cases, for example when the form $T = a\Delta - b\Delta^3$ has been used to represent part of the travel-time data. For purposes of illustration, the method is applied to determining the P velocity distribution for the whole region E of the Earth. The method incidentally illustrates anew the importance of the parameters ζ and α (defined below) in ray theory.

1. Introduction

The writer has previously shown (1945, 1959) that a velocity-distribution law of the form $v = ar^c$ is associated with a specially simple relation between travel times and distances for seismic rays. The purpose of the present paper is to show how this simplicity can be exploited to derive the actual velocity distribution from a set of given travel-time–distance data. The method to be exhibited provides an alternative to the classical Herglotz–Wiechert–Bateman method, and may often be more rapid and informative in practice.

Sections 1 and 2 will give the needed basic theory, and Section 3 will deal with special points in applying the theory. Section 4 will contain a few additional notes, and Section 5 will illustrate the use of the method to determine the P velocity distribution in the region E, the outer core of the Earth.

2. Preliminaries

The analysis will relate to families of rays all with end-points at a given level surface of a spherically symmetrical Earth. Let Δ denote the angular distance between the end-points of a ray, T the travel-time along the ray, and p the ray-parameter. Let v be the velocity at distance r from the centre of the Earth, and let

η, ζ and α be defined by

$$\eta = \frac{r}{v}; \quad \zeta = \frac{d\log v}{d\log r}; \tag{1}$$

$$\alpha = \frac{2}{1-\zeta} = \frac{2d\log r}{d\log \eta}. \tag{2}$$

Subscripts 0, 1 and Δ will denote values of variables at the given level surface, at the level $r = r_1$ (where $r_1 < r_0$), and at the lowest point of the ray whose distance is Δ, respectively.

In parts of the discussion, it will be convenient to use x and λ, defined by

$$x = r/r_0, \quad \lambda = p/\eta_0. \tag{3}$$

Use will frequently be made of the fact that

$$\eta_\Delta = p. \tag{4}$$

In an earlier discussion (1945, page 92, equations (7) and (8)), a form equivalent to

$$\frac{d\Delta}{dp} + \alpha_0(\eta_0^2 - p^2)^{-\frac{1}{2}} = \int_{\eta_\Delta}^{\eta_0} (\eta^2 - p^2)^{-\frac{1}{2}} \frac{d\alpha}{d\eta} d\eta \tag{5}$$

was derived.

In the particular case where ζ (and hence also α) is constant throughout the region of the Earth involved, the right side of (5) is zero, and (1) and (5) then yield

$$v = v_0 x^{\zeta_0}, \tag{6}$$

$$p = \eta_0 \cos(\Delta/\alpha_0), \tag{7}$$

and hence also

$$T = \eta_0 \alpha_0 \sin(\Delta/\alpha_0), \tag{8}$$

since

$$p = dT/d\Delta. \tag{9}$$

The simple unmodified relations (6), (7) and (8) give a good representation of (T, Δ) data and the associated velocity distributions in various parts of the Earth. It has been shown (1945, page 97), for example, that the relations fit the 1940 J.-B. tables for the entire region B within less than 1 s for P rays and 2 s for S rays. It will also be seen from Section 5 that they fit closely the travel-time data for the outer part of the region E.

The analysis to follow will show that, for those regions of the Earth where the travel-time data deviate significantly from the form (8), it is in general possible to take the form (6) as a first approximation to the velocity distribution, and apply fairly readily determined corrections. There are sizeable ranges of r in the Earth where the proportionate corrections are small. For such regions the method is particularly powerful because, first, only a small number of digits need be carried in the computations; secondly, interpolations where required can be carried out by simple proportion; thirdly, the trapezoidal rule suffices in numerical integrations.

3. Derivation of the basic formula

In the first approximation given by (6), (7) and (8), the values of ζ_0, η_0 and α_0 are the actual values of ζ, η and α at the level $r = r_0$. A formula will now be derived for the correction to be applied to α_0 to give the actual values of α at other levels. The derivation incidentally involves the solution of a form of Abel's integral equation fairly similar to that involved in the Herglotz method.

A key step is the introduction of the function β, defined by

$$\beta(\Delta) = \Delta - \alpha_0 \cos^{-1}(p/\eta_0), \tag{10}$$

to express departures from the simple form (7). From (9), it is seen that β is a directly calculable function of p or Δ when a (T, Δ) relation is given.

By (10), and then (4) and (5),

$$\frac{d\beta}{dp} = \frac{d\Delta}{dp} + \frac{\alpha_0}{\sqrt{(\eta_0^2 - p^2)}} \tag{11}$$

$$= \int_p^{\eta_0} (\eta^2 - p^2)^{-\frac{1}{2}} \frac{d\alpha}{d\eta} d\eta. \tag{12}$$

In (12), apply the operation

$$\int_{\eta_1}^{\eta_0} p(p^2 - \eta_1^2)^{-\frac{1}{2}} dp$$

to both sides. This involves integrating over a family of rays the lowest of which has parameter η_1 and the highest has $\Delta = 0$. We obtain

$$\int_{\eta_1}^{\eta_0} \frac{p}{\sqrt{(p^2 - \eta_1^2)}} \frac{d\beta}{dp} dp = \int_{\eta_1}^{\eta_0} dp \int_p^{\eta_0} \frac{p}{\sqrt{\{(p^2 - \eta_1^2)(\eta^2 - p^2)\}}} \frac{d\alpha}{d\eta} d\eta \tag{13}$$

$$= \int_{\eta_1}^{\eta_0} d\eta \int_{\eta_1}^{\eta} \frac{p}{\sqrt{\{(p^2 - \eta_1^2)(\eta^2 - p^2)\}}} \frac{d\alpha}{d\eta} dp$$

$$= \int_{\eta_1}^{\eta_0} \tfrac{1}{2}\pi \frac{d\alpha}{d\eta} d\eta$$

$$= \tfrac{1}{2}\pi(\alpha_0 - \alpha_1). \tag{14}$$

Integrating the left side of (13) by parts gives

$$\left[\frac{d\beta}{dp}\sqrt{(p^2 - \eta_1^2)}\right]_{\eta_1}^{\eta_0} - \int_{\eta_1}^{\eta_0} (p^2 - \eta_1^2)^{\frac{1}{2}} \frac{d^2\beta}{dp^2} dp. \tag{15}$$

When $p = \eta_0$, the right side of (12) is zero, so that $(d\beta/dp)_0 = 0$. When $p = \eta_1$, the expression $\sqrt{(p^2 - \eta_1^2)}$ is zero. Thus the square-bracket term in (15) vanishes.

Hence (14) becomes

$$\alpha_0-\alpha_1 = 2\pi^{-1}\int_{\eta_0}^{\eta_1}(p^2-\eta_1^2)^{\frac{1}{2}}\frac{d^2\beta}{dp^2}dp, \tag{16}$$

which is the basic formula required.

Suppose that a (T, Δ) relation, empirical or otherwise, is given. The values of η_0 and α_0 can then be deduced, and $d^2\beta/dp^2$ can, by (9) and (10), be determined as a function of p, or Δ. For any assigned η_1, (16) then yields the correction $\alpha_0-\alpha_1$ required to give α_1. Thus α is yielded as a function of η, and hence, by (1) and (2), v as a function of r.

4. The detailed procedure

4.1. *Determination of the first approximation.*—Assuming that T can, for small Δ, be represented as a power series in Δ, we can (1959, page 113) write

$$T = \eta_0\Delta - a\Delta^3 + O(\Delta^4), \tag{17}$$

and thence

$$\eta_0 = \lim_{\Delta\to 0}(T/\Delta), \tag{18}$$

$$p = \eta_0 - 3a\Delta^2 + O(\Delta^3),$$

$$\frac{dp}{d\Delta} = -6a\Delta + O(\Delta^2),$$

$$p^2 = \eta_0^2 - 6a\eta_0\Delta^2 + O(\Delta^3),$$

and

$$a = \lim_{\Delta\to 0}\left(\frac{\eta_0-p}{3\Delta^2}\right). \tag{19}$$

By (5),

$$\alpha_0 = -\lim_{\Delta\to 0}\{(\eta_0^2-p^2)^{\frac{1}{2}}\,d\Delta/dp\}$$

$$= (\sqrt{(6a\eta_0)}\Delta)(6a\Delta)^{-1}$$

$$= \sqrt{(\eta_0/6a)}. \tag{20}$$

The formulae (18), (19) and (20) enable the values of η_0 and α_0 to be deduced from the travel-time data. The first-approximation relations (6), (7) and (8), with the appropriate numerical coefficients inserted, can, if desired, be then immediately set down. In practice, in our method, it is, however, only the values of η_0 and α_0 that are necessarily required.

4.2. *Computation of the corrections* $\alpha_1-\alpha_0$.—It is convenient to take Δ as the independent variable, and, using (3) and (4), to re-write (16) as

$$\alpha_0-\alpha_\Delta = 2\pi^{-1}\int_0^\Delta(\lambda^2-\lambda_\Delta^2)^{-\frac{1}{2}}\frac{d\theta}{d\Delta}d\Delta,$$

$$\approx 2\pi^{-1}\sum(\lambda^2-\lambda_\Delta^2)^{-\frac{1}{2}}\delta\theta, \tag{21}$$

where

$$\theta = d\beta/d\lambda, \tag{22}$$

δ being here used to denote a finite step. (In these formulae, λ_Δ corresponds to α_Δ and to the integration terminal Δ, and is constant in each summation and integration.)

Towards evaluating (21), values of the independent variable Δ are first selected, defining intervals $\delta\Delta$ to which the $\delta\theta$ correspond. (In the illustration in Section 6, the values selected for Δ are 0°, 5°, 10°, etc.; this selection serves for "ordinary" regions (1959, page 112; see also Section 5.1).) Values of $\beta(\Delta)$ may be computed from the travel-time data by direct use of (9) and (10), or otherwise as shown in Section 4.4, and values of $\delta\theta$ then deduced.

The right side of (21) has to be evaluated separately for each λ_Δ, i.e. for each selected value of Δ. It is in general sufficiently accurate in evaluating the factors $(\lambda^2-\lambda_\Delta^2)^{-\frac{1}{2}}$ to compute values at the ends of each selected range $\delta\Delta$ and take simple means.

The summations in (21) can now be carried out, and yield a tabular relation between $\alpha_\Delta-\alpha_0$ and Δ. Call this relation . . . . . (23)

4.3. *Deduction of the velocity distribution.*—By (2) and (3),

$$d\log x = \tfrac{1}{2}\alpha\, d\log\eta,$$

or

$$d\log x = \tfrac{1}{2}\alpha_0\, d\log\eta + \tfrac{1}{2}(\alpha-\alpha_0)\, d\log\eta.$$

Using (4), we may then write

$$\left.\begin{aligned}\log x_\Delta &\approx \tfrac{1}{2}\alpha_0\log(p/\eta_0) + \textstyle\sum \tfrac{1}{2}(\alpha_\Delta-\alpha_0)\delta\log p\\ &= \xi(\Delta)+\mu(\Delta), \text{ say.}\end{aligned}\right\}. \tag{24}$$

The term $\xi(\Delta)$ is the first approximation to $\log x_\Delta$. (It is readily checked using (1), (2), (3) and (4), that, when $\mu(\Delta) = 0$, (24) yields (6).) The numerical values of $\xi(\Delta)$ are found immediately since α_0 and η_0 have been determined (alternatively, the values could be deduced using (6)).

The correction term $\mu(\Delta)$ is computed using the relation (23).

By this means x_Δ is found as a function of Δ, while $\eta_\Delta\ (= p)$ is already known in terms of Δ. Hence x, or r/r_0, is connected with η, or r/v, so that v is determined as a function of r.

The method is powerful because the first approximation ξ to $\log x_\Delta$ is obtainable after a few minutes' work from the raw travel-time data, while the correction term μ, being relatively small over wide ranges of values of Δ, can be computed with comparatively little trouble.

4.4. *Determination of θ for moderately small Δ.*—For moderately small Δ, Jeffreys and others have commonly represented travel-time data for particular regions of the Earth by the form

$$T = \eta_0\Delta - a\Delta^3. \tag{25}$$

The form (8) is already a close approximation to (25), and the practical question arises of deducing the needed corrections in the simplest way.

By reference to results in Section 4.1, we find that (25) entails

$$d\Delta/d\lambda = -\alpha_0^2/\Delta,$$

and

$$\begin{aligned}(1-\lambda^2)^{-\frac{1}{2}} &= \{1-(1-3a\Delta^2/\eta_0)^2\}^{-\frac{1}{2}} \\ &= \{1-(1-\Delta^2/2\alpha_0^2)^2\}^{-\frac{1}{2}} \\ &= \alpha_0\Delta^{-1}(1-\Delta^2/4\alpha_0^2)^{-\frac{1}{2}} \\ &\approx \alpha_0\Delta^{-1}(1+\Delta^2/8\alpha_0^2+3\Delta^4/128\alpha_0^4).\end{aligned}$$

By (3), (11) and (22),

$$\theta = \frac{d\beta}{d\lambda} = \frac{d\Delta}{d\lambda} + \frac{\alpha_0}{\sqrt{(1-\lambda^2)}},$$

so that, when (25) is given,

$$\theta \approx \tfrac{1}{8}\Delta(1+3\Delta^2/16\alpha_0^2). \tag{26}$$

Now α_0 is commonly of order unity. Hence, so long as (25) applies, the very simple formula $\delta\theta = \delta\Delta/8$ may, for values of Δ up to a considerable fraction of a radian, be used in applying (21) to determine values of the correction $\alpha_0 - \alpha_\Delta$.

This simplification will be used in Section 5.

5. Further notes

5.1 *Departures from ordinary behaviour in v.*—Except for a few limited ranges of r, the values of the variable ζ, which represents the ratio of dv/dr to v/r, are found to lie between about $-2{\cdot}5$ and zero for all observed P and S rays throughout the Earth. Regions throughout which ζ deviates but little from this range of values are called "ordinary" (1959, page 112). (There is no point in practice in setting down a more precise definition.)

Apart from the complication to be considered in Section 5.3, the method of this paper can be applied without modification for all ordinary regions of the Earth.

For regions inside part of which $\zeta > 1$, the variable α diverges to infinity, and the method formally fails. The failure, however, arises from the physical fact that in this case there exist ranges of depth inside which no ray has its lowest point. This fact affects all methods equally, and no mathematical device can side-step the failure.

For other types of deviation from ordinary behaviour, additional consideration has to be given to numerical detail, and 5° ranges of Δ will not be always suitable. The device of "stripping the Earth" is desirable in dealing with boundaries at which v or dv/dr is discontinuous. But for all such cases the needed modifications (1959, pages 112–118) again affect equally all methods of computing v.

5.2 *The singularity in v at r = 0.*—In the seismic context, ζ_0 is ordinarily negative, so that the law (6) entails that $v \to \infty$ as $r \to 0$. Since the writer's first publication (1945) of (6)*, some seismologists have held this singularity to be an objection to using (6), and have continued to use the more usual circular-ray

* Miss I. Lehmann informs me that (6) was previously used by A. Mohorovičić.

approximation, given by $v = a - br^2$, notwithstanding the very cumbrous form of the accompanying (T, Δ) relation (1959, page 86).

The present paper shows, however, that the singularity is not a source of difficulty in practice. First, neither the law (6) nor any other simple law is used in practice for any sizeable region of the Earth which includes the centre. Secondly, although v as given by (6) diverges at $r = 0$, it is α that is computed in our method, and in the first approximation α keeps the finite value α_0 for all r, including $r = 0$. For ordinary regions, the functions β and θ are likewise finite (for $\Delta = \pi$, which corresponds to $r_\Delta = 0$, β has the moderate value $\pi(1 - \frac{1}{2}\alpha_0)$); and the subsequent use of (21) to determine the corrections $\alpha_\Delta - \alpha_0$ is not touched by the singularity in the first approximation to v.

The point is illustrated in Section 6 where the method is successfully applied to obtain a velocity distribution over the wide range $0{\cdot}4 \leqslant x \leqslant 1$, corresponding to $0^\circ \leqslant \Delta \leqslant 120^\circ$. Inspection of tables in Section 5 will show, moreover, that if ordinary behaviour had held beyond $\Delta = 120^\circ$, then the velocities would have been equally readily determined for values of x appreciably less than $0{\cdot}4$.

5.3. For small x, there is, however, a real source of difficulty which does not arise from (6). This is that $p \to 0$ and $\log p$ diverges as $x_\Delta \to 0$, so that the terms in (24) become numerically large for small x_Δ, and a special problem in computation is presented. But once again the same problem arises in all methods. For example, in the Herglotz method, the basic formula (1959, page 119), which is equivalent to

$$\ln x_\Delta = -\int_0^\Delta \cosh^{-1}(p'/p)\, d\Delta' \tag{27}$$

(in (27), p' corresponds to Δ', and p to Δ and x_Δ), has in effect the same left side as (24), and the difficulty is encountered in precisely the same form.

The selection of the variables α, and then β, in our method is thus seen to avoid all trouble connected with the singularity in v in (6), while such further difficulties as there are arise unavoidably in all methods.

5.4. *An alternative to using the function* β.—Eliminating Δ between (7) and (8) gives the first approximation relation between T and p, namely

$$T = \alpha_0\sqrt{(\eta_0^2 - p^2)}. \tag{28}$$

Hence the function γ, defined by

$$\gamma(\Delta) = T - \alpha_0\sqrt{(\eta_0^2 - p^2)}, \tag{29}$$

is another form expressing departures from the first approximation.

By (9) and (11),

$$\frac{d\gamma}{dp} = \frac{dT}{d\Delta}\frac{d\Delta}{dp} + \frac{\alpha_0 p}{\sqrt{(\eta_0^2 - p^2)}}$$

$$= p\frac{d\beta}{dp}. \tag{30}$$

By (14) and (30),

$$\tfrac{1}{2}\pi(\alpha_0-\alpha_1) = \int_{\eta_1}^{\eta_0} (p^2-\eta_1^2)^{-\frac{1}{2}}(d\gamma/dp)\,dp$$

$$= [(d\gamma/dp)\cosh^{-1}(p/\eta_1)]_{\eta_1}^{\eta_0} - \int_{\eta_1}^{\eta_0} \cosh^{-1}\left(\frac{p}{\eta_1}\right)\frac{d^2\gamma}{dp^2}\,dp.$$

Like $d\beta/dp$, $d\gamma/dp$ vanishes when $p = \eta_0$, while $\cosh^{-1}(p/\eta_1)$ vanishes when $p = \eta_1$. Hence

$$\alpha_1-\alpha_0 = 2\pi^{-1}\int_{\eta_1}^{\eta_0} \cosh^{-1}(p/\eta_1)\frac{d^2\gamma}{dp^2}\,dp. \tag{31}$$

The formula (31) serves as an alternative basis to (16) for computing the corrections $\alpha_\Delta-\alpha_0$. By (29), $\gamma(\Delta)$ can be directly computed from the given (T, Δ) relation, using (9); and (31) can then be used analogously to (16).

For moderately small Δ for which the form (25) is adequate, (3), (22) and (26) give

$$\frac{d\beta}{dp} \approx \tfrac{1}{8}\eta_0^{-1}\Delta(1+3\Delta^2/16\alpha_0^2).$$

Also, by (9), (20) and (25),

$$p = \eta_0(1-\Delta^2/2\alpha_0^2).$$

Hence, by (30),

$$\frac{d\gamma}{dp} \approx \tfrac{1}{8}\Delta(1-5\Delta^2/16\alpha_0^2). \tag{32}$$

The simple formula $\delta(d\gamma/dp) = \delta\Delta/8$ may therefore be used analogously to the formula $\delta\theta = \delta\Delta/8$ of Section 4.4.

Thus with the use of γ instead of β, the corrections $\alpha_\Delta-\alpha_0$ can be equally readily computed.

5.5. The classical formula (27) is simpler in appearance than either (16) or (31), but our method derives its advantage because:

(i) The first approximation is yielded straight away and is a good approximation for many regions.

(ii) Attention is concentrated in the analysis on the character of the deviations from the first approximation. This applies especially to the variation of important functions such as α and ζ, as well as β and θ.

(iii) The total numerical labour in obtaining the corrections is small because of the small number of digits generally needing to be carried, and of the simplicity with which interpolations, numerical integrations, etc., may be carried out.

6. Application to the region E of the Earth

The method, in the version involving the function β, will now be applied to the *P* travel-time data used by Jeffreys (1939) for the region *E*. The region *E* has

been selected for purposes of illustration because it involves the largest range of values of r in the Earth for which the behaviour of v is ordinary. Cases where the behaviour is not ordinary are not considered in this paper, since each case presents its own problems. Moreover, in many cases, the problem is avoided by stripping the Earth.

The kilometre, second and radian will be taken as units except as otherwise indicated. The symbol b will denote the number of degrees, 57·296, in a radian.

6.1. *The empirical data.*—We take as starting point the data in Table 1 below, which were derived by Jeffreys after applying (9) to the raw (T, Δ) data for the Earth stripped of its mantle.

For $0^\circ \leqslant \Delta \leqslant 35^\circ$, the values in Table 1 fit the form (25) precisely. The formula used by Jeffreys is equivalent to

$$T = 7{\cdot}4787b\Delta - 0{\cdot}0002042b^3\Delta^3, \tag{33}$$

from which

$$p = 7{\cdot}4787b - 0{\cdot}0006126b^3\Delta^2. \tag{34}$$

For $\Delta > 35^\circ$, the values of p and Δ are not tied to a simple formula.

Table 1

The given data on p and Δ

Δ deg.	p/b	Δ deg.	p/b
0	7·4787	65	5·14
5	7·4634	70	4·86
10	7·4174	75	4·60
15	7·3409	80	4·36
20	7·2337	85	4·12
25	7·0958	90	3·88
30	6·9274	95	3·64
35	6·7283	100	3·38
40	6·49	105	3·11
45	6·21	110	2·85
50	5·92	115	2·58
55	5·65	120	2·31
60	5·40		

6.2. *The first approximation.*—Application of (18), (20) and then (2) to (33) yields

$$\eta_0 = 7{\cdot}4787b, \quad \alpha_0 = 1{\cdot}3636, \quad \zeta_0 = -0{\cdot}4667. \tag{35}$$

Taking the core-radius r_0 to be 3 473 km then gives

$$v_0 = r_0/\eta_0 = 3473/7{\cdot}4787b = 8{\cdot}1045.$$

Thus the first-approximation relations are

$$v = 8{\cdot}1045x^{-0{\cdot}4667}, \tag{36}$$

$$p = 7{\cdot}4787b\cos(0{\cdot}73335\Delta), \tag{37}$$

$$T = 584{\cdot}3\sin(0{\cdot}73335\Delta), \tag{38}$$

$$T = 1{\cdot}3636\sqrt{(55{\cdot}931b^2 - p^2)}. \tag{39}$$

Table 2 shows the velocity distribution as computed using (36), and also the distribution obtained by Jeffreys using the Herglotz method. The latter is the distribution we seek. Table 2 is set down to show the degree of approximation achieved in using (36), but is not a necessary part of the computations.

The necessary step at this stage is to compute the first-approximation values ξ of $\log x_\Delta$ in terms of Δ. By (24) and (35),

$$\xi = \tfrac{1}{2}\alpha_0 \log(p/\gamma_0) = 0{\cdot}6818 \log\lambda. \qquad (40)$$

The second column of Table 4 gives the values of ξ, computed using (40), against the corresponding Δ.

Table 2

First approximation to velocity distribution compared with the velocity distribution of Jeffreys

x	v (1st approx.) km/s	v (Jeffreys) km/s
1·00	8·10	8·10
0·98	8·18	8·18
0·96	8·26	8·26
0·94	8·34	8·35
0·92	8·43	8·44
0·90	8·51	8·53
0·88	8·60	8·63
0·86	8·70	8·74
0·84	8·79	8·83
0·82	8·89	8·93
0·80	8·99	9·03
0·78	9·09	9·11
0·76	9·21	9·20
0·74	9·33	9·28
0·72	9·45	9·37
0·70	9·57	9·44
0·68	9·70	9·52
0·66	9·84	9·58
0·64	9·98	9·65
0·62	10·13	9·72
0·60	10·29	9·78
0·58	10·45	9·84
0·56	10·62	9·90
0·54	10·80	9·97
0·52	11·00	10·03
0·50	11·20	10·10
0·48	11·42	10·17
0·46	11·64	10·23
0·44	11·90	10·30
0·42	12·15	10·37
0·40	12·43	10·44

5.3. *Completion of the solution.*—By (22), the evaluation of $\delta\theta$ involves taking second differences of λ or p. For this reason, values of p in Table 1 for $\Delta > 35^\circ$ were smoothed, by two successive applications of the smoothing formula

$$y = y^0 - \delta^4 y^0/12, \qquad (41)$$

before use in the computations, including the computation of ξ just referred to.

(Having regard to the comparative smallness of the sought corrections, this smoothing is possibly an unnecessary refinement, but (41) is so simple in application that little additional work is required.)

The next step is to compute $\delta\theta$ for use in (21). For $0 \leqslant \Delta \leqslant 35^\circ$, the formula $\delta\theta = \delta\Delta/8$ of Section 4.4 avails, and yields $\delta\theta = 0{\cdot}011$ for each 5° step in Δ.

For $\Delta \geqslant 35^\circ$, β was computed directly from (10), and the values of θ and $\delta\theta$ deduced. The results are shown in Table 3.

In deriving the θ, values of $\delta\beta/\delta\lambda$ were computed for each original 5° step in Δ, and simple means used to derive values at the ends of steps.

Table 3

λ, β, θ and $\delta\theta$ for $\Delta \geqslant 35^\circ$

Δ	$\lambda = p/\eta_0$	β	$\theta = d\beta/d\lambda$	$\delta\theta$
deg.		deg.	rad	rad
35	0·89966	−0·289	+0·076	
				+0·150
40	0·86721	−0·723	+0·225	
				−0·095
45	0·83034	−1·181	+0·130	
				−0·237
50	0·79225	−1·277	−0·107	
				−0·275
55	0·75605	−0·747	−0·382	
				−0·154
60	0·72158	+0·254	−0·536	
				−0·040
65	0·68651	+1·395	−0·576	
				−0·084
70	0·65043	+2·605	−0·660	
				−0·188
75	0·61553	+4·081	−0·848	
				−0·172
80	0·58267	+5·895	−1·020	
				−0·082
85	0·55091	+7·860	−1·102	
				0·000
90	0·51906	+9·914	−1·101	
				+0·061
95	0·48622	+11·946	−1·041	
				+0·071
100	0·45183	+13·927	−0·970	
				+0·037
105	0·41640	+15·830	−0·933	
				−0·018
110	0·38073	+17·792	−0·951	
				−0·024
115	0·34498	+19·793	−0·975	
				(−0·025)
120	0·30888	+21·810	(−1·000)	

Values of the factor $(\lambda^2 - \lambda_\Delta{}^2)^{-\frac{1}{2}}$ in (21) were directly computed, and (21) then applied to determine the corrections $\alpha_\Delta - \alpha_0$. The corrections μ to $\log x_\Delta$, given by (24) by

$$\mu = \sum \tfrac{1}{2}(\alpha_\Delta - \alpha_0)\delta \log p, \qquad (42)$$

were then deduced. In these operations simple means were again used in all interpolations.

The deduced values of $\log x_\Delta$, given by (24) as the sum of ξ and μ, are shown in the fifth column of Table 4. The sixth column gives the corresponding x_Δ, while v_Δ is deduced using

$$v_\Delta = (r/\eta)_\Delta = r_0 x_\Delta / p. \tag{43}$$

The required velocity distribution is given by the values of x_Δ and v_Δ shown in Table 4. Interpolation to correspond to the values of x in Table 2 is seen to give virtually total agreement with the velocities of Jeffreys.

The efficacy of the new method has thus been demonstrated.

Table 4

Completion of determination of velocity

Δ deg.	ξ	$\alpha_0-\alpha_\Delta$	μ	$\log x_\Delta = \xi+\mu$	x_Δ	v_Δ
0	·0000	·000	·0000	·0000	1·0000	8·10
5	−·0006	·000	·0000	−·0006	·9986	8·11
10	−·0024	+·001	·0000	−·0024	·9945	8·13
15	−·0055	+·003	·0000	−·0055	·9876	8·15
20	−·0099	+·006	·0000	−·0099	·9774	8·19
25	−·0156	+·008	·0000	−·0156	·9647	8·24
30	−·0227	+·012	+·0001	−·0226	·9493	8·31
35	−·0313	+·017	+·0002	−·0311	·9309	8·39
40	−·0422	+·031	+·0004	−·0418	·9082	8·49
45	−·0550	+·045	+·0007	−·0543	·8824	8·61
50	−·0690	+·027	+·0011	−·0679	·8553	8·75
55	−·0827	−·015	+·0012	−·0815	·8288	8·88
60	−·0966	−·034	+·0009	−·0957	·8023	9·01
65	−·1114	−·099	+·0002	−·1112	·7741	9·14
70	−·1274	−·138	−·0012	−·1286	·7437	9·27
75	−·1437	−·176	−·0030	−·1467	·7134	9·39
80	−·1599	−·218	−·0054	−·1653	·6834	9·51
85	−·1765	−·278	−·0084	−·1849	·6533	9·61
90	−·1941	−·302	−·0121	−·2062	·6220	9·71
95	−·2135	−·321	−·0166	−·2301	·5887	9·81
100	−·2352	−·338	−·0218	−·2570	·5534	9·93
105	−·2594	−·348	−·0279	−·2873	·5160	10·04
110	−·2859	−·362	−·0345	−·3204	·4782	10·18
115	−·3151	−·378	−·0425	−·3576	·4389	10·31
120	−·3479	−·392	−·0517	−·3996	·3985	10·46

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Research Note

Seismic Model Experiments on the Shape and Amplitude of the Refracted Wave

R. J. Donato

It has been known for many years from theoretical consideration that the first arrival part of a refracted seismic wave should have a waveform given by the time integral of the original wave (Jeffreys 1926). Quantitatively this is difficult to verify experimentally from field refraction records, although it is one of the causes of the "rounding" of the first part of the refracted waveform. However, by using seismic model techniques the experimental verification both of the wave shape and of the amplitude predicted by equation (1) becomes possible, and this note describes such experiments.

Liquid media were used throughout the experiments. An electric spark immersed in the medium overlying the refractor simulated the seismic explosive source, and a lead zirconate transducer horizontally separated from the source by distance R (see figure) received the refracted wave. The receiver was designed to have a flat frequency response throughout the spectrum covered by the pressure pulse. The relationship between the received pressure of the refracted wave $p_r(T)$ and the incident pressure $p_0(t)$ referred to unit distance from the source is given in terms of the elastic properties of the media by

$$p_r(T) = \frac{2c_2}{\left[\left(\frac{c_2}{c_1}\right)^2 - 1\right]\frac{\rho_2}{\rho_1}(RL^3)^{\frac{1}{2}}} \int_{\tau}^{T} p_0(t-\tau)dt \qquad (1)$$

where

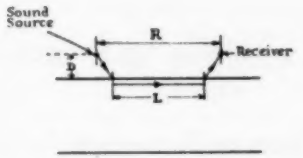
$$\tau = \frac{1}{c_2}\left[R + 2D\sqrt{\left(\frac{c_2}{c_1}\right)^2 - 1}\right]$$

D = the height above refractor of the source and the receiver.
L = the distance travelled in the refractor.
c_1, c_2 = the velocities of the overburden and the refractor.
ρ_1, ρ_2 = the densities of the overburden and refractor.

The figure shows photographic records of (*a*) the original pulse, (*b*) the pulse after passing through an electronic integrator, (*c*) the refracted pulse. The last waveform of the series gives the result of integrating (*a*) numerically. Curves (*b*), (*c*) and (*d*) compare favourably.

All the terms in equation (1) can be measured and substituted in the equation. The calculated and measured peak amplitudes of the refracted wave for two

liquid/liquid combinations agree to better than 5 per cent which is within the experimental accuracy.



(a) Direct pulse

(b) Direct pulse integrated electronically

(c) Refracted pulse

(d) Numerical integration of (a)

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Notes on Progress in Geophysics

Current Research in Geophysics in France

G. Jobert

1. Introduction

Geophysical research in France is conducted by very various teams. The following Institutes and Observatories of Physics of the Earth depend on a University: Paris (+Martinique and Guadeloupe), Strasbourg, Clermont-Ferrand, Pic du Midi, Alger (+Tamanrasset). The National Centre for Scientific Research has recently put in operation a Centre for Geophysical Studies in the Nièvre (at Garchy). Other observatories depend on the Office for Scientific and Technical Research Overseas (ORSTOM): M'Bour (Sénégal), Bangui (République Centrafricaine), Nouméa (Nouvelle Calédonie). But research in specific fields is also carried on either in universities or by public services as the National Meteorological Office, National Geographical Institute, Bureau of Geological, Geophysical and Mining Researches (BRGGM), or by private companies like the General Geophysical Company (CGG).

A French National Committee for Geodesy and Geophysics groups all geophysicists. We have used in preparing our report its yearly proceedings which include a complete bibliography of the work done. We will limit ourselves to presenting—without intending to be exhaustive—the principal researches undertaken in France from 1955 and the principal results obtained in the field of the physics of the Earth's interior.

2. Gravity

An absolute value of gravity was determined in 1958 (Å. Thulin 1960) by means of a method initiated by Ch. Volet, at the International Bureau of Weights and Measures in Sèvres. A scale of platinum–iridium is photographed as it falls. The precision of the measurement is ± 1 mgal and the value obtained is 12·8 mgal lower than that of the Potsdam system, in very good agreement with pendulum determinations by Clark and Agaletskii. An interferometric method is in preparation.

The French gravity network is now complete. It has been achieved through the collaboration of the Bureau of Geophysical Researches (BRGGM) which publishes the maps (scales of 1/200 000 and 1/80 000), the Geographical Institute and various companies. The lines of the unified European network of levelling have been specially surveyed. Numerous measures have also been made in overseas countries (North and West Africa, Sahara, Madagascar, Antarctica, Pacific French Islands).

Studies of crustal structure based on these measures have been made: in France by S. Coron, who examined in some detail the relations between isostatic

anomalies and the geology of mountainous regions as Brittany, Massif Central and Corsica; she found recently (1959) that the thickness of the crust in the western Alps does not follow the dissymmetry of the topography, its maximum being in the middle of the 1000 m generalized hypsometric line, rather west of the highest regions; in North Africa by J. Lagrula; in French West Africa by Y. Crenn, who showed (1956) that in the central part of this region the mean value of the isostatic anomalies is very small, and presented an hypothesis to explain the observed compensation. J. Goguel (1958) put forward arguments favourable to a regional isostatic compensation combined with rupture due to tension, to explain the characteristics of the Rhine graben. Some other anomalous zones could also be interpreted as due to the complexity of the crustal structure and not to a lack of isostatic equilibrium.

New concepts for the interpretation of the gravity anomalies have been presented (R. Perrin 1957). The upper part of the crust could have been modified by superficial processes as erosion, together with metamorphism and migrations in opposite directions of the heavier and lighter elements. Some difficulties of the classical theory of isostasy are thus avoided.

Theoretical researches on the geoid, Stokes's formula and the external gravity field are pursued by J. J. Levallois and V. Baranov. The late P. Lejay and S. Coron have calculated the absolute deviation of the vertical at three points (Paris, Nice, Guelt es Stel in North Africa) using isostatic anomalies. They estimated the precision of their results to be about $0''{\cdot}5$.

The tidal variations of the gravity vector have been studied especially from the beginning of the IGY. R. Lecolazet observed continuously the variations of the gravity intensity from 1957 in Strasbourg with modified North American meters. Four stations, each over 45 days, have been made in Africa and Madagascar. A La Coste–Romberg tide meter has been operated during several months in Paris and Strasbourg.

A precise method for computing amplitudes and phases of various harmonic components of the terrestrial tides and the corresponding theoretical factors has been given by R. Lecolazet. His principal results, in agreement with those from the La Coste–Romberg meter, are the following: the ratio of the observed amplitude to the theoretical one is greater for the semi-diurnal waves than for the diurnal ones (about 1·21 and 1·18). The phase differences are generally smaller than 1°. The wave S_1, with a 24 hours period, is practically nonexistent (1959).

The tidal variations of the vertical direction relative to the ground were observed from the beginning of 1958 in Paris and near the Pyrénées by means of silica tiltmeters built by P. A. Blum. These pendulums are of great sensitivity and stability (drift smaller than $0''{\cdot}05$ during several months). A similar 2-minutes period apparatus with a photocell and a galvanometer has been recently installed in Paris.

The perturbations of the terrestrial tides due to oceanic loads and thermal variations have been studied theoretically (G. Jobert 1960).

3. Seismology

The French seismological network includes stations in France (9) and in various overseas countries (17). Temporary stations have also been installed during the IGY in Antarctica and southern French islands. Macroseismic studies

and preliminary determinations of epicentres are carried out by the French Seismological Bureau in Strasbourg under J. P. Rothé's direction.

A book on theoretical seismometry will be soon published by F. Duclaux. The theory of the modern electromagnetic seismographs will be presented with a description of the principal types in use and methods for calibration.

J. Jolivet is now completing two sets of seismographs for the Antilles. Electrostatic amplification enables several response curves and various magnifications to be obtained with the same pendulum simultaneously for the study of near, regional or distant earthquakes. Other electronic seismographs have been used by Y. Rocard for detection of atomic bombs.

During the IGY seismic soundings have been made in Antarctica to measure the depth of the basement under the icecap (B. Imbert 1959). A programme for the investigation of Alpine structure has been prepared under the auspices of the European Seismological Commission. In France two series of experiments have already been undertaken with the collaboration of many French and foreign geophysicists. A first series of explosions was made in 1956 with charges from 1 to 10 tons in the folded zone of Briançon. The 1958 programme was realized in the Mercantour granite massif with charges from 1 to 25 tons. Thirty-one sets have occupied 149 refraction stations along 4 profiles, and 3 reflexion stations. The refraction seismographs had generally maximum amplifications between 2×10^5 and 5×10^6. A preliminary report on the 1956 experiment has been presented (P. Tardi 1957). The following velocities have been observed: 4·5 km/s in the superficial coal layer; about 5·4 km/s to a depth of 2 km and 6·07 km/s below. The Conrad discontinuity is probably appearing on E–W profiles corresponding to a depth of about 25 km under sea-level and a velocity of 6·7 km/s. The Mohorovičić discontinuity seems to appear but with a poorly defined hodochrone (velocity of 8 km/s). Owing to the international nature of the experiments the publication of the complete results is delayed.

Other researches on the crustal structure are undertaken in Paris under Mme Labrouste's guidance. In particular some anomalies in the propagation of body waves in a deep focus earthquake in the Hindu Kush have been studied. A new phase between P and pP has been considered as due to a reflexion at a discontinuity near the epicentre at a depth of about 75 km (M. A. Choudhury 1958).

In Strasbourg, E. Peterschmitt examined the seismicity of the Tyrrhenian sea. He found that it can be compared with that of a Pacific arc. The foci are distributed near a conical surface whose dip is greater than 60°. The study of some peculiarities in the propagation of seismic waves from Mid-Atlantic Ridge earthquakes is in progress at M'Bour.

Free oscillations of the Earth and dispersion of long-period Love waves are being theoretically studied by N. Jobert (1960). Surface and mantle waves showing dispersion have been observed by means of the long-period tiltmeters of P. A. Blum.

Seismic models are under investigation at the French Petroleum Institute for interpreting records obtained in seismic soundings (in particular models with varying densities and velocities). Syntheses of seismic records are made either by calculation or experimentally by means of an analogue model, to determine the influence of multiple reflexions. Similar researches are undertaken at the General Geophysical Company.

Researches on the microseismic agitation are pursued in Paris by P. Bernard.

4. Terrestrial magnetism

The variations of the geomagnetic field are continuously recorded in France at Chambon la Forêt, near Paris, and overseas at Tamanrasset, M'Bour, Bangui and Tananarive. Since the beginning of the IGY stations have been installed in Antarctica and on Kerguelen Island, and temporarily at Tahiti.

During the IGY the principal effort has been directed to the study of the short-period variations of the geomagnetic field by the use of uniform equipment (bar-fluxmeters of E. Selzer). At the Chambon Observatory special attention is now being given to the improvement of the knowledge of the absolute values of the magnetic elements. For this purpose a nuclear magnetometer of the Abragam–Solomon type (based on the "Overhauser effect") is now included in the regular equipment of the observatory. A few magnetometers of the same type used in France for various purposes are checked from time to time with this one. Besides a few other types of nuclear meters are under experimentation, specially one with optical pumping (A. Kastler).

At the Rock Magnetism Laboratory of the Institute of Physics of the Earth in Paris, studies on the properties of the various magnetizations of baked clays are pursued under E. Thellier's direction, with an application to the analysis of superimposed magnetizations. Studies on archaeomagnetism are continued using oriented samples of baked clays (declination and inclination from kilns and fire-places, inclination only from displaced objects as pots, bricks). The intensity of the past thermoremanent field is also determined (E. and O. Thellier 1959). Lastly palaeomagnetic researches on igneous rocks are undertaken, not specially to determine the positions of the magnetic poles in the past, but to help geologists and give them means to separate different lava flows. The influence of anisotropy in schistose rocks has been recently made evident.

In addition, researches on the magnetic properties of the superficial soil are pursued. They could be due to fires made on the ground (E. Le Borgne 1960). The fire effect on the soil susceptibility could be interpreted as a series of transformations of iron oxides, which could also occur in schists heated in convenient oxido-reduction conditions.

In Strasbourg A. Roche is pursuing his studies on the remanent magnetization of igneous rocks of various ages from Auvergne, Provence, North Africa and Madagascar. His new results (1960) on Miocene and quaternary rocks are in good agreement with those obtained in Japan on the hypothesis of a geocentric dipole far deviated from the present axis of rotation. The displacement of this dipole axis seems to have been considerable during relatively short times and one must be very cautious when interpreting some discrepancies in palaeomagnetic results as due to crustal deformations when the samples are taken in different regions and of uncertain relative ages.

5. Glaciology

During the IGY current measurements have been carried out by the French expedition in Terre Adélie. The results obtained during the various campaigns on the Greenland ice-cap have been the subject of several detailed publications by the French Polar Expeditions.

At the Laboratory of Applied Geophysics (University of Paris) an equipment comprising electrometric amplifiers has been developed and tested on the Aletsch glacier for the study of the simultaneous rapid variations of the magnetic and

telluric fields. It has been used to evaluate the thickness of the ice on the Saint-Sorlin glacier (L. Cagniard 1959).

Studies are in progress on the flow mechanisms of the glacier and the ice deformations at the Laboratory of Glaciology of the "Collège de France", where the propagation of electromagnetic waves in the ice is also under examination.

The mechanisms of a glacier have been examined by L. Lliboutry who introduced the scheme of a reservoir glacier in opposition to the classical scheme of an evacuating one for the Greenland ice-cap (1956). Other particularities of the glaciers, as blue bands, shear moraines and crevasses, have also found an explanation on the basis of theoretical calculations. The dynamic friction on the ground, which could bring an explanation of the pulsatory motion of the glaciers, has also been studied by Lliboutry.

6. Other branches

Zircons have been used by Mme Hée and her collaborators to determine the ages of various rocks. But in France studies on radiogeology, natural radioactivity and geothermy are generally undertaken in departments of geology. The repartition of radioactive substances in granitic massifs has been examined for example at the School of Applied Geology and Mining Prospection in Nancy, at the Laboratory of Applied Geology in Paris and at the Laboratory of Geology in Clermont-Ferrand.

No systematic measurements of the heat flow from the Earth are made in France. But a theoretical study on the influence of the surficial temperature on the geothermal flux, which could have been underestimated in measurements made in the permafrost, has been presented (J. Goguel) and the heat propagation around a spherical source buried in a homogeneous rock has been examined (P. Laffitte).

The Antillean volcanoes are continuously in observation. The Soufrière in the Guadeloupe Island had a crisis (October 1956) producing two types of seisms which could be attributed to ruptures and to the clearing phase.

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1960 *Janvier.*

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Geophysical Discussion of the Royal Astronomical Society

A discussion was held in the rooms of the Royal Astronomical Society on 1960 January 22 at 16h 15m. The subject was **The Form and Attenuation of the Seismic Wave.** In the temporary absence of the Chairman, *Sir Edward Bullard*, *Sir Harold Jeffreys* opened the discussion and called for the first speaker.

Dr E. R. Lapwood, of Cambridge University, delivered a paper on phase changes due to reflections in elastic wave propagation, and the resultant change in shape of a seismic pulse. When a plane wave struck a plane boundary between two elastic media, perfect reflection occurred beyond a critical distance determined by the critical angle. The reflected ray underwent a sudden change of phase which rose from 0 to π degrees as the complement of the angle of incidence decreased to zero from the angle determined by the ratio of the wave velocities in the two layers ($\cos^{-1}\beta'/\beta$). Thus there was a change of phase in each ray reflected beyond the critical distance. In a layer bounded above by a free surface and below by a layer of higher wave velocity, there was zero phase change for waves reflected beyond the critical angle at the free surface but a phase change for each such reflection from the interface. The use of Fourier's integral permitted one (subject to certain mathematical restrictions) to find the response to a disturbance, assuming the disturbance to be a periodic function of the form $\cos wt$. The response could be expressed as a combination of the source function and the allied function (as introduced by Lamb) of the disturbance, and by multiplying these together it could be seen how the appearance of the reflected or refracted pulse changed with distance from the source. This led in some cases to a response which began before the time of arrival as calculated from ray theory; but this never characterized a first arrival whose path was a minimum time path whereas the response discussed was a stationary time path. Though no experimental work was known of changes in pulse shape on passage through solid layers, Arons and Yennie had made observations of sound pulses passing through water. Their work used the sea bed as an interface between two liquids; the pulse shape was taken as a sharp rise followed by an exponential fall; and the change in pulse shape with distance could be calculated and compared with the results of experiment. Slides were shown to illustrate these points. The speaker ended by briefly reviewing the work Jeffreys and he had done on phase changes in the neighbourhood of a caustic for the case of a source within a liquid sphere where differences in the arrivals of *pP* and *PP* were noted. He stressed that phase changes were common in elastic wave propagation and many phenomena caused the change in shape of a wave pulse.

The Chairman began the discussion by asking the speaker if he would comment on the interest this might hold for talks at the Geneva disarmament conference where interest might be expected to centre on impulses whose initial movement was radial. Lapwood replied that phase changes could be expected and that the possibility of several impulses in this case could give further complications. *Dr Stoneley* of Cambridge University commented that with anisotropic material not only compressional waves would be generated. In an anisotropic elastic

medium there was no longer a clearcut distinction between purely compressional and purely distortional waves. Sir Harold Jeffreys remarked that, on the whole, the arrival times of the *P* phases in near earthquakes behaved reasonably well, but those of *Sn* and *Sg* showed variable anomalies of one or two seconds. It would be interesting to have some explanation of these.

The second speaker, *Mr G. Bradfield* of the National Physical Laboratory, considered the three processes governing the form and attenuation of the seismic wave: the launching, attenuation and pick-up of the signal. The purpose of the contribution was twofold. First by broad considerations the frequency spectrum at a distance was obtained and secondly the possibility of the setting up of a shear wave by an explosion was examined.

The analysis of a simple dilating spherical radiator by Blake, Pursey and others, yielded an effective radiation impedance for a cavity which, for the lower frequencies, acted as a mass reactance in series with a compliance and a radiation resistance, the impedance of the compliance predominating. Since the radiation resistance varied as the square of the frequency, this network favoured the radiation of the higher frequencies. In terms of the cavity pressure wave, the radiated power increased as the fourth power of the frequency. If the explosion was short and sharp, the cavity pressure had a frequency spectrum independent of frequency; while if it were a step function, its frequency spectrum varied in amplitude inversely as the frequency. When the radiated wave had travelled some distance, the influence of an exponentially attenuating term in the propagation equation, with index varying as the first or second power of the frequency, led to a great attenuation of the higher frequency components of the spectrum. The overall frequency spectrum was very much as though the system contained a band-pass filter the width of which, relative to the central frequency transmitted, remained fairly constant but the central frequency decreased with range. Of the two cases, linear and quadratic law of frequency in the index, the latter should have led to a faster variation with range of f_{max}, the frequency of maximum response, than the former. Yet some experimental evidence (McDonal & others) appeared to show a fall-off of f_{max} still slower with range than Ricker's theory would predict for the quadratic law. This theory also led to a narrower frequency band ($3\frac{1}{2}$:1) than the above simple theory would predict or than McDonal & others actually found (about 6 for the ratio of upper to lower frequencies at $\frac{1}{2}$ amplitude points). It was perhaps desirable to point out that the situation was more complicated than the above simple picture described since the explosion shattered the rock near the cavity. Nevertheless, the main effect of this seemed to be that the theory merely needed to assume a rather larger cavity and a measure of additional attenuation around it. The speaker's, and the majority of other published data on the change which took place in the attentuation as the frequency was changed supported the view that the attenuation per wavelength was constant over a very wide range of frequency, at least in dry rocks. As the dispersion was negligible, the attenuation index was linear in frequency. This dependence was typical of fine-grained metals and many other solids and it was customary to explain this in terms of the presence of loosely anchored dislocations. The fact that this loss was at least 10 000 times as great in sandstone as it was in α-quartz was surprising but might be attributed to the porosity and the stress-raising this caused in impurities at the grain boundaries; these impurities were likely to be highly strained and lossy due to dislocations caused by thermal stresses. When water was present the decrement of the sandstone could be increased by a further 10 times. The frequency law for the

index, which was two for a normal liquid, could be intermediate between one and two for a water-logged sandstone, although it was probably rarely much above unity. This contrasted sharply with the theory of Ricker, who assumed a quadratic law, but agreed well in some respects with the propagation data found by McDonal & others.

The radiation of shear waves from an explosion was of interest because shear waves had more likelihood of resulting in an anomalous start to the received wave than did longitudinal waves. This was impossible from a spherical cavity in an isotropic medium but the customary bore-hole differed in two respects both of which led to shear wave radiation. These were: first that the cavity was elongated and secondly that it was weak axially at the entrance end. A third manner in which shear waves could be generated, i.e. by the delayed but progressive firing of the explosion down the charge, was not considered. The radial strain for a given pressure at the spherical end of an elongated hole was only half the strain remote from the end. This difference along the length amounted to a shear strain and accordingly shear waves were radiated. Since, in addition, the entrance end was weak axially, the strain there was less still, and the above tendency was enhanced. The maximum of the radiated shear wave was at about 45° to the hole. Just as with a longitudinal wave, a shear wave only needed one mode change to invert it but since the latter at 45° incidence yielded a large longitudinal wave, and since the maximum shear wave was radiated at about 45° to the hole, an energetic anomalous explosion wave was quite likely in certain circumstances.

Dr P. N. S. O'Brien, of the British Petroleum Company, spoke next and dealt with the seismic disturbance due to an explosion and transmitted through sedimentary rocks. Recordings were shown which established that the predominant part of the initial disturbance due to an explosion in homogeneous rocks normally lasted between one and ten milliseconds. Conventional seismic prospecting records were shown which emphasized that normally the ground motion due to an explosion lasted several seconds. This lengthening of the initial disturbance was produced largely by reflection and refraction at the numerous velocity contrasts which existed in the Earth's crust. In reflection shooting, computation showed that the decay of seismic energy with time was almost entirely due to the expansion of the wavefront and to straightforward reflection losses at horizontal acoustic contrasts. Absorption, scattering and the like, played only a small part which was most easily shown in the gradual decrease in the frequency content of the seismic record with time. In refraction shooting the seismic wave travelled a considerable distance within a single rock. From measurements on the decay of the seismic pulse with distance from the shot, estimates could be made of the absorption and scattering losses. These usually amounted to between 0·1 and 1·0 db/wavelength and these values were consistent with those measured in the laboratory. In sedimentary rocks, which were usually water saturated, it was not yet established whether the absorption per unit distance was proportional to the first or to a higher power of frequency. For dry rocks it seemed clear that it was proportional to the first power. Examples were shown as slides which emphasized the difficulties encountered when making field measurements of absorption.

In the discussion following this paper, *Dr T. F. Gaskell* drew attention to the "tail" which followed some *P* waves shown in O'Brien's slides and said these had also been observed on records made by Bullard and Gaskell. In order to obtain a single pulse from a hammer blow on an anvil fixed in the ground, it was necessary to put the detector (geophone) a few inches from the anvil; at greater distances the

"tail" appeared. This was explained as due to multiple waves produced by a strong increase of velocity with depth in the surface layer of the ground. *Dr A. H. Cook* compared the transform expressing reflections at a diffraction grating with that for seismic waves at velocity discontinuities and suggested that it might be possible to make some progress in the problem by studying the convolution function at the detection point. *The Chairman* said that one could get striking results by applying an electric filter to the detected signals or by numerically convolving the results. *Dr D. C. Tozer* asked the frequency of the ultrasonic waves and Dr O'Brien replied that they were in kilocycles. *Mr Bradfield* asked if detonating more explosive material would improve the signal to noise ratio. O'Brien said this was so in the case of wind noise. The Chairman said this was not a help when the noise arose from elastic waves generated by ditches, etc., near the explosion. Bradfield suggested that sinking the geophones might also be effective. The pattern of shots could be used to decrease surface noise in reflection prospecting techniques, said Gaskell, and more explosive helped in refraction shooting. Signal to noise ratio was not a major problem in reflection shooting, he added.

The fourth speaker was *Dr R. Stoneley*. He considered the problem of propagation of surface waves in an anisotropic medium. It was not possible to give a general answer as it was in the case of isotropic media, to the question of whether or not Rayleigh and Love waves could exist. He spoke of the distinction made by Jeffreys between *SV* and *SH* waves and of work by Musgrave, both matters bearing on the problem of anisotropy. Considering an hexagonal crystal as an example of a transversely isotropic substance, the problem could be brought to a consideration of two waves rather than the three required for a completely anisotropic substance. The mathematics could be treated as in standard vibration theory. There was no possibility of clearly separating the elastic waves into compressional and shear waves. Both Rayleigh and Love waves were developed in this hexagonal case and these combined into composite wave forms. Some experimental studies by Levine, using data obtained from boreholes, indicated that a transversely isotropic model better fitted his results than did a completely isotropic model.

In the discussion which followed, *Dr O'Brien* asked if the transverse ground motion which could be observed when detecting refracted waves with three-component geophones might be due to anisotropy. *Dr Stoneley* replied that, while anisotropy might explain this, an affirmative answer could not be assumed as other possible factors were operating in this rather complex problem.

The Chairman thanked the speakers and those who participated in the discussions and adjourned the meeting at 18 h 05 m.

Book Reviews

The Earth

Its Origin, History and Physical Constitution

Harold Jeffreys

(4th edition, 1959, pp. 420+xvi, Cambridge University Press)

All geophysicists will be grateful that Sir Harold Jeffreys has found opportunity to revise his classic work. Those hitherto unfamiliar with *The Earth* will discover with delight Sir Harold's elegant and powerful command of the English language and of mathematical exposition. They will join the many readers who have been convinced by the arguments, amused by the chapter headings, roused by the polemics, and thrilled by the triumphs of scientific method as the author's story is unfolded.

Those to whom the earlier editions are familiar will be most concerned to see what changes have been made in the Fourth Edition compared with the Third. In plan and paragraph numbering there is almost no alteration, but Sir Harold has added material from the research of the years 1950–58, including his own, on travel-times of earthquakes, and on the modified Lomnitz law of creep, with its implications for studies of the departure of the Earth's mantle from purely elastic behaviour. There are also new considerations on polar wandering, the Moon's surface features, and the Earth's thermal history. The bibliography has been considerably augmented, and includes about 700 references, in general running as far as 1956.

Readers will not be surprised to find that on the whole Sir Harold continues to hold, and to defend with vigorous and lively argument, his well-known views on the constitution of the Earth's crust, mountain formation, continental drift, and the Earth's thermal history.

It is easier to mention topics omitted than to list all those included. This book does not discuss the information on geophysical problems which has lately been derived from nuclear explosions and the motion of artificial satellites, nor recent work on continental and oceanic structure as deduced from widespread and accurate observations of surface waves. But in spite of the great range of modern developments in geophysics, there is still very little which cannot be found treated with insight in this book. The reader cannot fail to be impressed by the scope of Sir Harold's enquiries and the extent of his information.

The arguments are close-knit, and the mathematics, never included for its own sake, is usually advanced and sometimes severe. But the text is so written that the argument can be understood when the mathematical details are taken for granted.

The first edition of *The Earth* was published in 1924. In the second, published in 1929, a great deal of new material was included. The third edition, in

1950, incorporated the results of rapid advances in all branches of geophysics, and again differed greatly from its predecessor. But the fourth edition, changed mainly in details from the third, shows that the accepted body of theory and experimental data is now large. To that body Jeffreys's 120 original papers and books have made a major contribution.

Sir Harold's close association with the development of geophysics over forty-five years has given to his writing, with its flair for highlighting key historical discoveries, something of the excitement of a first-class detective story. He must be a dull reader who does not catch from it some of the exhilaration of the intellectual search and struggle which Sir Harold describes with enthusiasm, and in which he has engaged with such outstanding success.

E.R.L.

Physics of the Earth's Interior

Beno Gutenberg

(International Geophysics Series Vol. 1, pp. 240, Academic Press)

Professor Gutenberg, whose death recently occurred, spent a lifetime studying the Earth, and as a seismologist was eminently qualified to write a book on the Earth's interior. This book makes an admirable start to a new series on geophysics and should make a good textbook for graduate students.

The study of the Earth's interior relies most heavily on the results of seismological research but, to a lesser extent, draws on a host of other subjects. In these days, when the effort expended on geophysical research is so enormous, no one person can expect to have more than a nodding acquaintance with a few of these fields. Hence, there arises the need for a book which will contain all the important information from a variety of subjects and which supplies enough references to make the consultation of the original papers a relatively easy matter. In these respects, this book is very good. It is, generally speaking, well up to date with references to papers published during 1959. Perhaps it was expecting too much to see references to the new data on the figure of the Earth obtained from Earth satellites, but such subjects as radiative heat transfer, phase changes and the palaeomagnetic data collected during the past few years are discussed.

A consistent feature of the book is the emphasis given to the physical problems involved, with mathematics reduced to the minimum necessary for exposition. Separate chapters are devoted to the crust, mantle and core and other chapters are concerned with the physical conditions and rheological properties of the various regions. Where the author feels less than expert, he is content to quote, sometimes with a healthy scepticism, extracts from original papers. This attitude is particularly evident in the first few pages of the book on methods of investigation and the accuracy of results. He draws attention to the dangers of using the results of investigations without a proper appreciation of their limitations. So often have the "facts" about the Earth's interior been shown to be without foundation. For this section alone, the book deserves a place on the shelves of geophysicists.

D.C.T.

Physics and Chemistry of the Earth, Vol. 3

Ed. L. H. Ahrens, F. Press, K. Rankama and S. K. Runcorn

(1959, pp. 464 + viii, Pergamon Press)

This is the third volume of an annual series of review articles on topics in geophysics and geochemistry. It contains eight articles varying in length from 17 to 93 pages. The standard of interest and importance is naturally variable; some articles are reviews of published work and some give large amounts of new data.

The first article is a brief discussion of palaeoclimates by J. Wyatt Dunham. He emphasizes the increasing uncertainty in the climatic interpretation of the fossil evidence as times more remote from the present are considered. The main conclusion is that before the Tertiary ice ages the climate was usually warmer than it is at present. This is ascribed to a tropical climate covering a wider range of latitude than it does at present, but it could equally well be explained by a movement of the continents to higher latitudes.

C. B. Officer & others, in a long paper, review the seismic work that has been done at sea around the Eastern Caribbean. This paper gives a mass of detail not published elsewhere, including 56 pages of travel-time plots. No other island arc is so well explored, and these results must have a great influence on theories of the origin of such features. It is found that the deeper parts of the Venezuela Basin have a structure similar to that of the Atlantic Ocean outside the arc, but with a rather deeper Moho and a lower seismic velocity below it. Under the line of the arc of islands and of the Aves swell, which does not reach the surface, the Moho falls to greater and unknown depths.

C. L. Drake, M. Ewing and G. H. Sutton discuss the continental margin of the east coast of America north of Cape Hatteras. A great deal of detailed information is given, but unfortunately the intelligibility of this important work is marred by the omission of Figure 35 from all copies available to the reviewer; this figure should give the sections setting out the main results of the work.* The chief conclusion is that the offshore sediments are divided by a ridge into two basins. The inner basin is compared with the valley and ridge province of the Appalachians before folding took place (a miogeosyncline) and the outer basin to the Piedmont between the Appalachians and the sea (eugeosyncline).

A. G. W. Cameron discusses the origin of the elements. The subject has developed rapidly during the last few years; observation has shown large differences in the abundances of some elements in different types of stars. On the theoretical side a great variety of processes has been investigated and a good semi-quantitative explanation of many features of the abundances can now be given by assuming the elements to be synthesized from hydrogen in the interior of stars and during super-nova outbursts.

E. Roedder reviews the published work on the equilibrium of silicate melts of petrological interest at atmospheric pressure and in the absence of water.

J. A. S. Adams & others discuss the geochemistry of thorium and uranium. They consider the distribution of these elements in rocks and meteorites and the geochemical cycles through which they go. Ore deposits are excluded from the discussion.

* Since writing this I have received a copy of the missing figure from the Lamont Observatory.

S. I. Tomkieff lists and classifies 1 200 papers on geochemistry, published in Russia between 1954 and 1956, and provides brief comments on some of them.

D. C. Tozer writes on the electrical properties of the Earth's interior. He describes the work of Chapman and his collaborators on the separation of short-period magnetic fluctuations into parts of external and internal origin and the conclusions that can be drawn about the radial variation of the electrical conductivity. He then considers how this variation can be accounted for and concludes that the mantle is an intrinsic semi-conductor. If the mantle is composed of magnesium-rich olivine a temperature of 4 500 °K is required at the boundary of the core. This review brings together the applications of solid state physics to the estimation of the electrical properties of the Earth's interior and should help to increase interest in a rather neglected branch of geophysics.

E.C.B.

Les Transformations de Mellin et de Hankel (Applications à la Physique Mathématique)

S. Colombo

(Centre National de la Recherche Scientifique, Paris, 1959, 99 pp., 10 NF.)

This is one of a series of monographs from the *Centre d'Études Mathematiques en vue des Applications*. The author derives properties of Mellin and Hankel transforms from those of Laplace and Fourier transforms. Ch. I gives a general introduction to integral transforms, Chs. II and III deal with Mellin and Hankel transforms respectively, Ch. IV gives a selection of applications to the solution of partial differential equations, and Ch. V discusses integral equations of mixed type.

Throughout the book, the author states carefully the conditions of validity of formulae and theorems, but does not aim to give rigorous proofs. He does outline formal derivations so as to enable the reader to see the pattern of development of the subject or technique under discussion. He includes seven tables of transforms.

The book suffers somewhat from over-compression: the examples in Ch. IV are not adequate to show either the full details of technique or the particular advantages of the transforms there utilized. But it forms a handy summary of the properties of Mellin and Hankel transforms and should prove useful to theoretical geophysicists.

E.R.L.

Geochemical Methods of Prospecting and Exploration for Petroleum and Natural Gas

A. A. Kartsev, Z. A. Tabasaranskii, M. I. Subbota and G. A. Mogilevskii

English translation edited by P. A. Witherspoon and W. D. Romey

(Berkeley and Los Angeles: University of California Press; London: Cambridge University Press, 1959. 94s.)

Natural exudations of oil and gas at the surface of the Earth have been known in various places throughout historical times, and during the last hundred years, important oilfields have been developed near many of them. In the search for oil,

these surface indications have always been considered valuable clues, but as the more readily discoverable fields were found, exploration has come to depend increasingly on geophysical investigations, which at best indicate no more than the presence of favourable structures. The test of drilling shows all too often that while some structures contain oil, many do not. It has also become clear that much oil may occur in traps which have little structural expression, and which are consequently hard and expensive to find by geophysical means. The industry has always hoped for the development of some technique capable of indicating directly the presence of oil at depth, and not a few inventors have claimed a success for their instruments which experience has not confirmed.

The first national attempt to solve this problem seems to have been made in the late twenties in Russia, where V. A. Sokolov put forward the view that the rocks above an accumulation of oil or gas could not completely prevent the slow upward passage of light hydrocarbon gases. He developed an apparatus to determine minute quantities of such gases in the air entrapped in near-surface soil, and made surveys which showed the existence of concentrations of hydrocarbons over oilfields. The method was tested in Russia on a large scale in the mid-thirties, apparently with some success. Practical and theoretical difficulties, however, came to light, but these served to stimulate research into many aspects of petroleum geochemistry. The book under review records the state the subject had reached in Russia by 1954; its translation marks the renewed interest in this form of exploration in U.S.A., where relatively little use had been made of geochemistry in the face of competition from established geophysical procedures.

The work opens with a chapter on the geochemistry of petroleum formations, which touches on the composition and physical properties of petroleum, its variability, and on the influence of the entities—rocks, waters, and organisms—that are in contact with it in the Earth. The third chapter discusses at some length the geochemistry of gases. Between them comes an outline of geochemical prospecting methods, which are classed as direct when they detect petroleum hydrocarbons as gas or bitumen, and as indirect when biological or hydrochemical techniques are employed. Subsequent chapters discuss each method in turn, with some account of the operations and apparatus used, and with examples of the results. The difficulties that occur in practice are not minimized. A short chapter on the role of geochemical methods in the whole context of exploring for petroleum rounds off the work; it stresses that all available geological and geophysical data must be considered in interpreting geochemical surveys. There are bibliographies to each principal chapter, largely, but by no means exclusively, of Russian works.

What is the outlook for such methods of exploration? At first sight, the apparent simplicity of those which utilize gas or bitumen is appealing. Granted the possibility of upward migration, the occurrence of marked concentrations of these substances in the soil should indicate an oilfield. Further acquaintance shows the difficulties; for instance, gas surveys repeated over the same ground at different times of year may give strikingly different pictures; syngenetic bitumens are widespread in many rocks and most soils; bacteria may flourish in the soil and destroy hydrocarbons. Anomalous distributions of hydrocarbons, or other geochemical indices, undoubtedly occur, and some seem to be related to oilfields. It does not appear that hydrocarbon surveys have yet been extended over frankly unpromising igneous or metamorphic terrains. The results of such an experiment would be interesting, and might be surprising. Perhaps the most hopeful

development at present lies in the application of capillary column vapour phase chromatography to the study of soil hydrocarbons.

The book is reproduced from typescript, and misprints are quite few. The diagrams are of unequal merit, particularly the sketch-maps, and there is no index. To anyone engaged in petroleum exploration who is brought up against the limitations of geophysical methods, it can be warmly recommended as stimulating reading.

R.W.

Geophysical Journal

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